Domestic electricity generation using waterwheels on moored barge

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2005
DECLARATION

I …Zoë Jones…………………….., confirm that this work submitted for assessment is my own and is expressed in my own words. Any uses made within it of the works of other authors in any form (e.g. ideas, equations, figures, text, tables, programmes) are properly acknowledged at the point of their use. A full list of the references employed has been included.

Signed: ................

Date: ……27th May 2005……………………….
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Abstract

This dissertation aims to design a waterwheel for very low head sites in rural Scotland to provide domestic electricity supply. The dissertation uses MathCAD to design a stream wheel and Excel to design the vessel that the wheel is fixed to. Options for the gearbox and generator are briefly considered.

During research it became clear that one of the major challenges facing waterwheel development is a lack of scientific analysis and the general perception that waterwheels are anachronistic and irrelevant. To illustrate the potential of waterwheels three case studies are included; a new wheel in the developing world, a mill renovation in the UK and an old wheel that still powers a working mill.

Keywords: undershot waterwheels, stream waterwheels, basic barge design, basic catamaran design, microhydropower,
**Glossary of Terms**

Archimedian Screw - age old technology used to lift water from lower heights to irrigate fields. Currently being developed in reverse to produce electricity. Patent owned by Ritz Attro (www.ritz-attro.de)

Gantt Chart - graphical representation of the stages of a project against the estimated time taken to complete each stage.

PERT Chart - chart showing chronological order of the stages of a project, illustrating which stages rely on previously completed components.

Kyoto Protocol - a summit on climate change held in Kyoto in 1997 saw 160 nations sign up to this Protocol laying down targets for greenhouse gas emissions reduction.

Undershot wheel - a waterwheel powered by the kinetic energy of water running below the centre, which pushes the blades around.

Overshot wheel - water enters the wheel near the top and falls into buckets, turning the wheel using the potential energy of the water.

Breastshot wheel - similar to an overshot wheel the water enters about halfway up the height of the wheel.

MathCAD - computer program allowing users to input equations, functions and matrices, alter variables and perform mathematical processes such as integration, differentiation etc. quickly

Head - the difference in the water height either side of the waterwheel

Zuppinger or Poncelet wheel - undershot wheel with curved paddles that use both kinetic energy and, by channelling the water into curved blades potential energy. This leads to higher outputs and greater efficiencies.
Glossary of Terms

Renewable Obligation (Scotland) Bill- government bill introduced in 2002 forcing electricity suppliers to buy 10% of their supply from renewable sources

Critical flow- point at which the flow of the water changes from being streamline to turbulent, signalling the use of different equations, theories and testing procedures

Supercritical flow- beyond the critical flow here flow is turbulent with a Froude number of 1

Froude Number- ratio of the force on an element of fluid to the weight of the element. In mathematical terms \( Fr = \frac{velocity \times (gravitational \ cons \ tan \ t \times \ characteristic \ length)}{0.5} \)
Nomenclature

\( \pi = \text{Pi} \sim 3.14 \)

\( \cos = \text{Cosine function} \)

\( x = \text{Angle of blade arm at centre of waterwheel relative to the vertical} \)

\( x_L = \text{Angle of blade arm at the centre of waterwheel relative to the vertical at the point of the blade leaving the water.} \)

\( x_1 = \text{Angle of blade arm at centre of waterwheel at point of blade beginning to leave the water} \)

\( \text{Current} = \text{Speed of the water horizontally} \)

\( V_c = \text{Component of the current acting in the blade’s direction} \)

\( V_b = \text{Rotational velocity of the blade} \)

\( V_r = \text{Relative velocity of blade} \)

\( C_d = \text{Coefficient of drag} \)

\( f = \text{Width of blade} \)

\( d = \text{Depth of blade} \)

\( L = \text{Distance from centre of waterwheel to bottom of blade} \)

\( y = \text{Distance from centre of waterwheel to top of blade} \)
Nomenclature

a(x) = Diagonal distance from centre of waterwheel to water line

D(x) = Depth of blade in water

A(x) = Surface area of blade in water

p = constant representing the proportion of velocity the blade picks up from the current

t = Time taken for one blade to complete one revolution

Larm = Lever arm from centre of waterwheel to the centre of the submerged area of blade

F(x) = Force on one blade moving from a vertical position to the point that the blade leaves the water

M(x) = Moment at waterwheel centre created by blade moving from a vertical position to the point the blade leaves the water

WorkDone = Work done on one blade from entering to leaving the water

N = Number of blades

TotalWorkDone = Total work done for entire wheel from entering to leaving the water

PowerAbsorbed = Total amount of power absorbed by wheel during one revolution

Side A = breadth of barge multiplied by the depth of the barge

Side B = length of barge multiplied by depth of barge
Zoë Jones

Nomenclature

q = distance from point of triangular barge end to the nearest side of barge rectangle

side F = breadth of the watertank multiplied by its depth

side E = length of the watertank multiplied by its depth
1 Introduction

1.1 Recent concerns over global warming and an over reliance on fossil fuels have led to an increased political, academic and public interest in renewable energy. The Kyoto Protocol sets strict standards for countries to limit their carbon emissions and research alternative energies.

1.2 Three viable areas of renewable energy have emerged- solar power, wind power and hydropower. This dissertation focuses on hydropower for a single domestic supply in rural Scotland.

1.3 Microhydropower refers to the production of 300 kW or less using turbines, waterwheels or Archimedian screws (1). With its low head and flow requirements, relatively low cost, “fish friendly” slow rotation, and ease of construction the waterwheel is experiencing a revival. They are especially relevant to small residential projects where the long payback period of turbines is prohibitive (1) and for developing countries, where maintenance and fabrication has to be simple. Three case studies are included in the literature review here to demonstrate waterwheels in use today.

1.4 This dissertation evaluates the typical energy usage of a hypothetical three bedroom household of two adults and two children and then models the dimensions and properties of the waterwheel required to produce this output. The wheel is to be situated on an open small river where there is no head difference and the flow velocity is approximately 2ms⁻¹.

1.5 The waterwheel is fixed to a type of barge or catamaran. This allows the wheel to move with the change in water level insuring a reliable power output and removes the cost of building a separate channel or weir. It is assumed that the waterway is not travelled on by boats.

1.6 Scotland has been chosen as there is a plentiful supply of water and rainfall and a number of island and highland communities, who would benefit from small scale power generation, as connection to the grid is expensive and unreliable.
2 Literature review

Fig. 1 Diagram showing process of research involved in the Literature Review
2.1 **Government Initiatives on Renewable Energy**

The UK government pledged under the 1997 Kyoto Agreement to cut greenhouse gas emissions by 12.5\% during 2008-2012 leading to a 20\% reduction on 1990 emission levels by 2010, in 5 years time (16). According to the 2003 Energy White Paper “Our energy future- creating a low carbon economy” the government aims for a 60\% reduction in greenhouse gas emissions by 2050. (16).

2.1.1 In 2003 only 2.7\% of the total energy used came from renewable sources. The Energy White Paper aims to increase this figure to 10\% by 2010. This will mean approximately 10,000 MW of electricity from renewable sources, which is roughly equivalent to 3,000-5,000 wind turbines (16).

2.1.2 In Scotland the Renewables Obligation (Scotland) bill was passed in 2002 forcing the licensed electricity suppliers to begin investigating renewable energy sources and purchasing a percentage of their energy from renewable sources (16). This will hopefully lead to electricity suppliers offering higher buy-up prices for renewable energy, making small scale renewable installations able to pay for themselves.

2.1.3 The Scottish Climate Change Programme signs Scotland up to generating 18\% of its energy from renewable sources by 2010, increasing that to 40\% by 2020 (17), with a range of measures, new bodies and funding to promote energy efficiency, renewable energy research and raise public awareness.
2.2 **Energy Usage and Renewable Energy Types**

Man requires energy at a basic level for heat and light. In primitive times burning wood provided warmth, cooking facilities, light and a place of gathering. Over the last 2000 years man has developed more sophisticated machinery for heating, lighting and entertainment; however the burning of fossil fuels has continued leading to the current situation of climate damage and few alternatives to fossil fuels.

2.21 In the latter part of the last century concern over the rapidly depleting fossil fuel supplies focused world leaders to meet and set CO₂ emission limits. Renewable energy is defined as energy that “is derived from an inexhaustible (wind, sun, sea) or replaceable (waste products, crops) source” (12).

2.22 Inexhaustible resources include:

- **Wind Power**- differences in temperature across the globe cause differences in air density setting up winds that can be used to turn turbines generating electricity.

- **Solar Power**- the sun’s energy can be used in photovoltaic cells, where a reaction causes electricity production or in a thermal or solar air heating system where water or air are warmed for heating.

- **Hydropower**- the hydrological cycle draws water from the sea into clouds, releasing rain onto high ground that then flows back to the sea along rivers, streams and tributaries. The flow of this water can be harnessed to turn the blades of a turbine, waterwheel, or the motor of an Archimedian Screw (1)

2.3 **Hydropower**

At the smaller end of the scale hydropower is classified as:

- **PicoHydropower**- up to 10kW
- **MicroHydropower**- 10kW-300kW
- **Small Hydropower**- 300kW-1000kW
- **Mini Hydropower**- above 1000 kW

(Taken from 13)

Within this dissertation the intention is to power one domestic dwelling in the UK making the power banding microhydropower.
2.21 Currently supplying only 0.8% (1456 MW) of the total energy consumption in the UK. Hydropower has been much underrated, with hydroelectric capacity in this country being estimated at 4,244 MW. (14) The map below (Fig. 2) shows the high concentrations of hydroelectric installations in Scotland and Wales, areas with greatest head differences (15). Large scale dams are seldom built now as they are costly to maintain and their construction has huge environmental impacts, such as water diversion, altering of river slopes and infrastructure creation, all of which disturb aquatic ecosystems (18).

![Map of Hydroelectric Plants in the UK as of end 2004. The dark blue stars signify sites that have been renewable obligation certified and light blue stars are yet to gain certification. (Taken from 15)](image-url)
2.4 **Microhydropower**

No clear estimates are available for microhydropower potential in the UK but some experts point to the 20,000 abandoned weir and watermill sites across the UK that could produce between 600 MW and 10,000 MW of power (2). Both the Department of Trade and Industry and the Scottish Executive seem to acknowledge the role that microhydropower has to play-

"If small-scale hydroelectric power from all of the streams and rivers in the UK could be tapped, it would be possible to produce 10,000 gigawatt hours... per year- enough to meet just over 3 per cent of our total electricity needs and making a significant contribution to the Government’s renewables target....” Department of Trade and Industry, 2005 (14)

"There are.... an increasing number of proposals for small run of river hydro projects and these projects,.....will ensure that hydro will continue to play its part in Scotland's renewable energy mix.” Scottish Executive Business and Industry, 2004 (19)

2.41 Yet despite this acknowledgement there has been little encouragement or official research into microhydropower since the “Small Scale Hydroelectric Generation Potential in the UK” Report by the Department of Energy in 1989. Frustratingly this only considered sites of more than 25 kW potential (and so ruling out waterwheels whose average power rating is 17.1kW) and with head differences of over 2m, immediately ruling out undershot and breastshot waterwheels (10). The report also discounted remote sites, with no grid connection as being uneconomical. These are exactly the kind of sites, where connection to the grid is too costly, that would benefit from the reliable independence that microhydropower can provide.

2.42 The publication of “The Layman’s guide on how to develop a small hydro site” in 1997 by the Commission of European Communities (21) did promote microhydropower to the public indicating costs, environmental impacts and basic site evaluation methods. This type of document has encouraged a whole group of “do-it-yourself hydro developers” (22) who have experimented independently with different heads, waterwheel types and generation systems.
2.44 Microhydropower can be generated using:

- **Turbines** - Water is funnelled into enclosed systems of blades rotating about the x or y axis. (See figs 3,4,6) Turbines have been well researched and developed yielding high efficiencies, but are still high in cost and complex to manufacture.

- **Waterwheels** - an age old technology where water enters at either the top or middle or it acts along the base. Waterwheels rose in popularity during the industrial revolution, but declined as electrical power took over from mechanical power, and have remained largely ignored ever since.

- **Archimedian Screw** - used for hundreds of years as a motor to raise water from lower fields for irrigation. Recently there has been renewed interest as the screw can be reversed running water from above, through the screw, turning a motor and generating electricity. Sparse experimental evidence exists but efficiencies are estimated at 70-80% (1).
2.5  Waterwheels

History of the Waterwheel

2.51  The inventor of the waterwheel is unknown but the undershot wheel is described by Vitruvius in 27BC (8). Initially the waterwheel was used to lift water and irrigate fields but was later used as a means to generate mechanical power for milling. The rapid industrialization of the Middle Ages led to an increase in waterwheel usage- 5000 mill sites are recorded in the Domesday Book of the 11th century (2).

2.52  Interest in the waterwheel continued from the 12th to the 20th century (2) reaching a peak in the 19th century. In England in 1850 there were 25-30,000 waterwheels in operation, in Ireland at the same time there were 6,400 and in Germany there were 40,000. (5) Many of the wheels constructed were built using rules of thumb passed down rather than specific scientific analysis. Texts such as “Water or Hydraulic Motors” (1894) by Philip R. Bjorling (7) contain rough formulae and tables for estimating the number of buckets, the theoretical velocity and the power output.

2.53  However the waterwheel was an important energy source that began to attract scientific interest. In 1752-54 John Smeaton built scale models of undershot and overshot wheels to test their efficiencies (8). The French government offered large rewards for more efficient designs, leading to the Poncelet design that was later refined and patented by Zuppinger (8). New roles were devised for the wheel such as on steam paddle ships where fuel was burnt to evaporate water, and create steam, turning the paddles, pushing the boat forwards. (9)

2.54  Waterwheels continued to be used well into the last century; in Bavaria in Germany 7,554 wheels were operating in 1927 (1), and in Switzerland nearly 7,000 small scale hydropower stations were being used up to 1924 (4). However the “lack of strong and reliable gearing systems, coupled with the advent of steam power and the introduction of higher speed water turbines rapidly led to the demise of the waterwheel” (3). In the UK waterwheels were left to rust or were removed, weirs were forgotten about or destroyed, populations moved away from the streams to the city and working knowledge of waterwheels was lost.
2.56 **Advantages of Waterwheels**

- Simpler technology than turbines lending themselves to the developing world for local fabrication and maintenance.
- Fish will hopefully be able to pass through waterwheels unharmed (more research is needed on this) and expensive fish screens will not be necessary (23)
- Faster pay back periods than turbines and in some cases Archimedian screws (1)
- Unlike wind turbines there seems to be less public resistance to waterwheels as they are not so out of place in the countryside for example the virtual wheel in Fig. 7. In fact the public seem to be interested in waterwheels judging by the number of visitor centres that proudly posses one.

![Fig.7 A typical weir with a virtual wheel installed (1)](image)

2.57 **Disadvantages of Waterwheels**

- The slow rotation of waterwheels (6-10 rpm) leads to high gearing ratios when trying to generate AC Electricity at 600-1500 rpm. More research is needed into different gear boxes and configurations.
- Waterwheels do produce a “low frequency thumping noise” (25) which is not well understood and could cause complaints. It is thought that altering the blade shape to a “spoon-shape” would lessen the blow on impact whilst maintaining a high drag coefficient. This would be better investigated by experiment.
- A lack of knowledge in the engineering profession. Few model experiments have been carried out on waterwheels and there is still much unknown about the flow, efficiency and physical properties of waterwheel
- If wheels are fixed to the side of the river then changes in flow level will cause fluctuations in power output making them unreliable. If the wheel is fixed to a barge or catamaran then a build up of river bed silt or a reduction in the water level could lead to the blades becoming damaged hitting the river bed. Some sort
of high frequency measuring device would be needed on the barge to check the
distance between the wheel and the river base, with regular maintenance
checking wear and tear on the blades.

2.58 In this dissertation turbines and the Archimedian screw have not been
investigated due to a lack of head at the site. Types of waterwheel include:

- **Overshot wheels**- \(2.5 \text{m} < \text{Head} < 10 \text{m}, \text{Flow} < 0.2 \text{ m}^3 \text{ s}^{-1} \text{ per m width}\) (1). Water enters above the wheel and falls into buckets turning the wheel with efficiencies of possibly 85% (1)

![Fig. 8 Overshot wheel (24)](image)

![Fig. 9 Undershoot Zuppinger wheel (24)](image)

- **Breast shot wheels**- \(1.5 \text{m} < \text{Head} < 3 \text{m}, 0.3 < \text{Flow} < 0.65 \text{ m}^3 \text{ s}^{-1} \text{ per m width}\) (1). Water enters half way up the diameter of the wheel, falling into buckets turning the wheel. Recent experiments at Queens University, Belfast indicate efficiencies of up to 87% (23), making them a viable option for low head sites.

![Fig. 10 Breast shot wheel(24)](image)

![Fig. 11 Working principle for breastshot wheel(24)](image)

- **Undershot wheels**- \(0.3 \text{m} < \text{Head} < 2.0 \text{m}, 0.45 < \text{Flow} < 1 \text{ m}^3 \text{ s}^{-1} \text{ per m width}\) (1). Some models use a very small head drop and curved blades to take potential energy from the river (60-77% efficiency (24)), others use the kinetic energy of the river on the blades (only 33% efficiency (8)).
2.6 **Types of Undershot Wheels**

Although considered inefficient even in the Industrial Revolution the undershot waterwheel continued to be manufactured as they could be sited on small streams in flatter areas, nearer to centres of population (8). There are several types of undershot wheel:

2.6.1 **Zuppinger wheel**

Designed by Walter Zuppinger and patented in 1883 this wheel uses only the potential energy of the river making it more efficient. The blades are curved carrying the water down a curved channel from a small weir and releasing it most efficiently, with minimal losses at the entrance or exit.

Fig. 12 above Sketch of installed Zuppinger wheel (27)

Fig. 13 right Same installed Zuppinger wheel (26)

Although sparse experimental data exists for all wheels Zuppinger wheels have perhaps been the most investigated. In 1979 students at the Technical University in Stuttgart, Germany tested an existing Zuppinger wheel that had been running since 1886. The test determined the flow rate and power output for a speed of 4.85 rotations per minute (rpm) and two flow rates of 1.48 and 3.1 m$^3$ s$^{-1}$. For $Q/Q_{\text{max}}=0.5$ the efficiency reached 77% and for $Q/Q_{\text{max}}=1$ efficiency reached 71% (24).

Figs 12 and 13 show a recently installed Hydrowatt Zuppinger wheel of diameter of 6.5m, a width of 2.3m, using a head difference of 1m to produce an electrical power output of 20kW at an overall efficiency of 70% (10). German based firm, Hydrowatt have built and installed 15 Zuppinger wheels with heads of 1-2.2m and power outputs of 4-45 kW from 1993 to 2001(10).
2.62 Impulse or Stream Wheels

Fig.14 Diagram of midstream wheel ((7) pg 18)

Considered the least economic these large diameter wheels have flat paddles immersed in the flow and use the kinetic energy of the current, requiring zero head.

As this is less than the potential energy the stream wheel is often regarded as being inefficient, however interest has resurged in them as “their application does not constitute a major change of river” (25). No civil works are required, the wheel can be moored on a barge or fixed to the side, and the relatively simple dimensions and layout mean that it can be constructed and repaired locally, lending itself more to the developing world than the Zuppinger wheel.

The Universities of Southampton and Berlin TU have joined together to test a 500mm stream wheel shown in Fig. 15 (25) Testing is due to start this month and will investigate the design characteristics such as the power output against speed, the overall efficiency and differences in the upstream and downstream depths, with a view to building an actual stream wheel in Munich. Data on that stream indicates that the flow is supercritical at 5ms\(^{-1}\) with the depth being only 0.5m (39).

Fig. 15 Stream wheel model at Berlin Technical University, 2005 (53)
2.63 Stream wheels in rectangular channels

If an undershot wheel is positioned very near the base of the river bed, and is nearly as wide as the channel then the power output and efficiency will be increased as the flow is forced through a small space at high velocity, becoming critical.

An early investigation into this phenomenon was the Cairo University based paper “Design of momentum water wheels used for mini hydropower” published in 1985 (28). The test set up an undershot waterwheel close to the base of the river bed with a sluice gate beside it. Water is forced through the narrow opening under the gate, increasing the speed as it passes some of its kinetic energy to the wheel, then settling at a lower height and slower speed downstream (See Fig.16 below). The paper estimated that efficiencies of up to 63% were possible with this design.

![Fig. 16 Cross Section of wheel and sluice gate with water flow and energy lines marked](image)

A more conservative outlook was expressed by a Japanese paper (30) in 2001 where an undershot waterwheel was placed 3mm from the test bed and power output was measured by simply attaching weights to the wheel until it no longer turned. The tests were very comprehensive altering the upstream height, Froude number and blade heights finally concluding that 45% efficiency could be obtained provided the dimensions of the channel and flow lay within set parameters.

![Fig.17 Cross section of rectangular channel experiment with undershot waterwheel](image)
An Austrian engineer has shown interest in this style of wheel developing his own untested “Staudruckmaschine” (45). Roughly translated as “hydropressure machine” the wheel is shown below.

The development of these types of wheels is hampered by their strict requirements of narrow shallow streams with high flows, smooth beds and sides (reference 30 sites irrigation channels as being suitable for these wheels), that are uniform enough to position a wheel in. The accuracy of construction and installation make them unsuitable for developing countries, and they would need a large trash rack to keep out debris. No study has been undertaken into how fish would navigate the wheel but with high velocities, and faster blade rotation this wheel could have a negative effect on aquatic life.

Fig. 18 (above) Photo of Staudruckmachine installed (45)
Fig. 19 and 20 (below) End view and cross section of wheel showing water (45)
2.7 Choice of waterwheel type

Seeing as the hypothetical site has no head difference a Zuppinger wheel would be unsuitable. The stream wheels fixed in rectangular channels require uniform channels with smooth beds and sides, which are not common in Scotland. So it was felt that a stream wheel would be most relevant to this dissertation requiring no head and being easy to fabricate.
2.8 Case Studies

Whilst researching this dissertation it became clear that waterwheel usage is not only held back by the lack of research and experimental evidence, high gearing ratios, etc. but also a perception amongst engineers and the public that an old technology could have no relevance to the present day. To help change this attitude the following case studies are included to illustrate scenarios where waterwheels have been installed or are used successfully.

2.81 Heatherslaw Mill, Ford and Etal Estates, Northumberland

Heatherslaw is a working mill grinding flour on the River Till in Ford and Etal Estate in the north east of England (see Fig…). The site has milled flour using hydropower since 1830 and is currently run by miller Mrs. Julia Nolan. The mill uses an undershot wheel 5m in diameter and 1.52m wide. Through a series of cogs, wheels and stones the rotating force of 20 horsepower (with sluice gate is fully open) transferred into mechanical power grinding grain into flour. The current wheel was constructed in the mid-1970’s from wood partly because it is easier to replace and to also prevent sparks in the highly flammable mill.

Although the miller admitted that a Zuppinger-style wheel would be more efficient and produce more power the mill could not risk the loss of revenue in time spent waiting for specialist servicing or replacement part production.

So one and a half wheels were constructed locally (the half being for spare parts), and the wheel is maintained locally.

Fig. 21 right Heatherslaw Mill (Authors own, 2005)

The river Till is well known locally for having a fast flow, aided by its sand gravel river bed. The half dam on the mill side (seen in Fig. 22) was actually installed to hold water in reserve in case of low rainfall. Water is drawn just in front of the dam into a covered channel running under the bank. The channel turns about 90 degrees (causing a drop in velocity), passes the trash rack and then enters the mill.

Fig. 22 River bank and half dam (Authors own, 2005)
Controlling the power output is important in order to mill different grains to different flour densities and so a sluice gate in front of the wheel controls the area of flow. There is also a sluice gate behind the wheel preventing back flow from storm surges jamming the wheel. At 10 rpm the wheels rotation is relatively slow allowing eels to pass through easily swimming to breeding grounds nearby.

Ford and Etal Estates receive a growing number of visitors each year to the mill and are keen to power the mills’ lighting (it has no heating) with renewable energy. A research project with Newcastle University has concluded that using the wheel’s outflow, and the considerable head drop there to turn a turbine would prevent altering the complex milling machinery and would raise visitor’s awareness of another type of microhydropower. (37)

2.82 **Howsham Mill Project, Renewable Heritage Trust, North Yorkshire**

Sitting on an island in the river Derwent in Howsham this former watermill was been abandoned as a derelict shell (see Fig. 25 next page). The mill was built in about 1770 by the eminent architect John Carr of York in the Gothic Revival style of fussy details that was seldom used on functional buildings. This lead to Howsham mill being described as “a building of maximum historical interest” by an inspector for the Royal Commission of Historical Monuments some 40 years ago (31).
The mill closed in 1947 and despite its Grade II listing and presence on several buildings at risk registers the mill fell into disrepair and dereliction, the roof fell in and parts of the undershot waterwheel were stolen (31).

Fig. 24 Howsham Mill c.1945 (32)  Fig. 25 Howsham Mill before renovation (32)
A local charity called the Renewable Heritage Trust have formed to restore the mill, turning it into an educational centre about renewable energy and to reinstate a waterwheel to power the centre, giving the public a chance to see renewable energy close up (32). The Trust relies on grants and donations and the hard work of their volunteers to carry out much of the renovation work giving the project a feel of community involvement (See Fig. 26).

A research project with Gerald Muller at Queens University has been carried out over 2004-2005 to determine a suitable wheel design. From his tests into breast shot wheels and their efficiencies the Trust are hoping to install a breast shot wheel designed by him. The site has plenty of water and the centre only needs a small output (approx 0.5kW) and so feel that a breast shot wheel, being smaller in diameter than an undershot wheel, would rotate faster limiting gearing ratios and being cheaper to construct. There are also plans to install an Archimidian hydraulic screw in the sluice channel which would enable the Trust to sell some power back to the National Grid funding, further rebuilding. (38)

Fig. 26 Community renovation of Howsham Mill (46)
2.83 **Pedley Wheel Charitable Trust, Sri Lanka**

The Pedley Wheel Charitable Trust began in a similar grass-roots renewables vein as the Howsham Project. In 1991 the trust built a demonstration overshot waterwheel in Pedley Wood in Cheshire to interest young people in renewable energy. After many attempts at solving the high gearing ratios the trust replaced the original tractor gearing systems they had with a chain configuration and an industrial gearbox (33). The trust also run their wheels as quickly as possible to minimise the torques and gearing ratios using about 65% of the power theoretically possible (34).

![An example of a Pedley Wheel (34)](image)

In 1998 the trust visited Lower Amanawela in the high up Southern Uplands of Sri Lanka. Although situated at approximately 460 m above sea level (35) the village did have two discarded irrigation channels, one 5m below the other, providing an ideal low head site. The trust then designed the 3.5m dia wheel in the UK, and had it built in Colombo whilst villagers built the powerhouse and civil works. To many critics surprise the wheel has worked well since installation producing 2.75 kW to light and heat 25 homes. There is also a stand alone gearbox allowing mechanical power for milling, woodwork, rice hulling etc. The electricity is administered by the Village Electricity Consumer’s Society who maintain the system and issue each house with a time slot when they can exceed their allocated 100W usage (35).

Since this project the trust has completed four waterwheel projects in Sri Lanka and are planning four more (36) providing power to a diverse range of properties including houses, community centres and computer training facilities.
3 **Technical Background**

3.11 **Waterwheel Design**

Unlike overshot and breast shot wheels a stream wheel does not rely on a difference in head (See Fig. 28) but instead uses the potential energy of the water to generate power.

![Fig. 28 Overshot wheel using potential energy and head difference to generate electricity](image)

3.12 **The Force on one paddle of a stream wheel** is given by rearranging the drag equation to get:  
\[ F = 0.5 \times \rho \times Cd \times A \times V_r^2 \]  
where \( \rho \) = Density of Water, \( Cd \) = Drag Coefficient, \( A \) = Area of blade in the water and \( V_r \) = velocity of the blade relative to the velocity of the water. MathCAD sheet can be viewed on enclosed CD

3.13 **Determination of Cd**

The drag coefficient for any object is influenced by its dimensions and journey through a fluid. From experimental data a variety of standard drag coefficients are known for set shapes. No experimental data exists on the drag coefficient of a waterwheel blade so here 1.5 has been used as an estimate, as this is the Cd value for a rectangular plate.
3.14 **Determination of A**

The area of the blade in the water obviously changes as the blade rotates about the central axis. To model this change mathematically the movement from a vertical position in the water to 90 degrees to the vertical is modelled. So if $x =$ the angle at the wheels centre the blade’s movement is modelled from $x = 0$ through $x = x_1$ (when the blade begins to leave the water) to $x = x_L$, (when the blade fully leaves the water). DraftT is the distance between the wheels centre and the top of the water, $y$ is the distance from the wheels centre to the top of the blade and $a(x)$ is the vertical distance from the wheel centre to the waterline.

![Diagram showing the passage of one blade through the water](image)

Using basic trigonometry $a(x)$ can be expressed:

\[
\cos \theta = \frac{adj}{hyp} \\
\cos x = \frac{\text{DraftT} + a(x)}{\text{hyp}}
\]

Re-arranged to get: $a(x) = \text{DraftT} \times \cos x$

If $a(x) \leq L - d$ then the paddle is still fully submerged and this area is given by:

\[
A(x) = d \times f \text{ where } f = \text{width of the blade}
\]

However as the angle $x$ increases towards $x_1$ (when the blade begins to leave the water) $a(x)$ increases and the submerged area decreases: $A(x) = (L - (a(x)) \times f$
### 3.14 Determination of $V_r(x)$

In order to express the relative velocity, $V_r$ it is necessary to know the velocity of the blade, $V_b$ and the velocity of the current in the direction of the blade, $V_c$. $V_c$ is found by using trigonometry:

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin(90 - x) = \frac{\text{Current}}{V_c}$$

$$V_c = \text{Current} \times \sin(90 - x)$$

$$V_c = \text{Current} \times \cos(x)$$

![Diagram showing the current component in the blade’s direction, $V_c$](image)

The velocity of the blade, $V_b$ is equal to the Current speed multiplied by a constant, $p$ that represents the proportion of the Current the blade should absorb for maximum power output.

A way to think of $p$ is to imagine the amount of work the water needs to do to turn a blade. If the wheel is allowed to rotate freely with the speed of the current then the water will easily push the blades around transferring no energy from water to blade—“freewheeling”. If the wheels speed is less than the current then the water will have to push harder to turn the blades the same distance transferring energy to the blade.

If, however the wheels resistance increases too high then the water will find it too difficult to move the blades, instead flowing around and underneath them, leaving the wheel stationary.
This phenomenon was first described by Antoine Parent in 1704 who correctly identified that the optimum value for $p$ was $1/3$ (40). This has been since proved by M. Denny in “The efficiency of overshot and undershot waterwheels” (8). In that paper the constant $p$ is renamed $c$ and the Power Output and Power Input are calculated. Efficiency = Power Output/Power Input and through cancelling out common terms, $p$ is found to be $1/3$ and the maximum efficiency, 33% (8).

Further evidence is provided in J. Wolfram’s derivation in Appendix B and testing using the MathCAD worksheet on page…

In order to maintain this blade speed the generator would be calibrated to provide a set amount of resistance. Some waterwheel owners prefer to run the wheel faster than $1/3$ of the current speed, in order to limit the gearing ratios involved, even at the expense of some efficiency (34)

So in conclusion,

$$V_c = V_c - V_b$$
$$V_c = (Current \times \cos x) - (p \times Current)$$

### 3.15 Moment at the Waterwheel Centre

The moment at the waterwheel centre is given by:

$$M(x) = F \times \text{LeverArm}$$

Where LeverArm = distance from the centre of the wheel to the centre of the submerged areas of the blade.

The work done between $x = 0$ and $x = xL$ is equal to the integral of moment at the wheel centre. The work done by one blade from entering the water to leaving is given by:

$$\text{WorkDone} = 2 \times \int M(x) \, dx$$

As the work done is measured in terms of radians not degrees a conversion factor of $\pi/180$ is used and the work done by the entire wheel is found using:

$$\text{Total Work Done} = N \times \text{Work Done} \times (\pi/180)$$
3.2 Basic Barge Design

In order for any object to float the downwards force that its weight supplies must equal the upward force supplied by the water. Archimedes found this upward force to be equal to $\rho gV$ where $\rho$ is the density of the fluid displaced (usually water), $g$ is the gravitational constant and $V$ is the volume of the object underwater. This seems logical when considering cruise liners with hundreds of rooms, swimming pools, restaurants etc. that balance their large weight with many storeys being below the waterline.

However in order for a barge to float it also has to be stable. Too much swaying from side to side may lead to water entering and the barge sinking or capsizing.

The Centre of Gravity (CG), that the ships weight acts through must be in the same vertical line as the Centre of Buoyancy (CB), which is in the centre of the underwater volume. Swaying from side to side is can be caused by the CG moving if one side is heavier than another, this in turn will cause the CB to move to line up vertically with the new CG position. Both centres can be defined using 3 co-ordinates but for stability the Vertical Centre of Gravity (VCG) and Vertical Centre of Buoyancy (VCB) are used.

VCG represents the height that the entire weight of the ship acts through, normally measured from the base.

$$VCG = \sum (\text{MassesOfComponents} \times \text{IndividualCentreofGravity}) \times \sum \text{MassesOfComponents}$$

VCB is simply given by $\text{Draft}T \div 2$, and in this case the draft is decided by the waterwheel’s requirements. In most barge design standard codes will be set as to the appropriate draft depth.

If a barge sways to one side due to wave action the Centre of Gravity will remain in the same place and the Centre of Buoyancy will move as the volume of barge underwater changes. If a vertical line is drawn from the original CB through CG upwards, intersecting a line from the new CB point they will meet at a theoretical point above the barge called the Metacentre, M. The distance BM is calculated using $I_{xx} \div V$, where $I_{xx}$ is the second moment of area of the waterplane from the centre line axis (for a rectangular shape $= bd^3 \div 12$).

The Metacentric height, GM is given by $VCB + BM - VCG$ and is the standard value used to determine a vessel’s stability. An iterative process is used firstly inputting dimensions similar to previous designs, balancing the applied forces and then insuring GM is within acceptable limits.
4 MathCAD Model

The following flow chart describes the design process for the waterwheel model. Design is very much an iterative process of refining and feeding in new ideas as they come to light.

- 4.1 Basic MathCAD model created
- 4.2 MathCAD model reviewed and modified
- 4.3 Variables tested to find most influential
- 4.4 Estimation of average domestic house energy requirements
- 4.5 Waterwheel dimensions determined and rotational speed checked

Fig. 32 Design process for waterwheel
4.1 Basic MathCAD Model

Using the theory explained in the Technical Background a basic MathCAD model was created describing the movement of a blade from the vertical position to the point when it leaves the water (see enclosed CD for copy). A copy of this early model appears on the next five pages with user inputs highlighted in red and observations highlighted in blue.

Initial assumptions included:

- The angle at which the blade leaves the water, \( x_L \) is determined by the designer as 45 degrees.
- The area of the blade submerged decreases proportionally as the angle, \( x \) increases.
- The constant, \( p \) will be kept at the optimum 1/3 by later generator selection and design.
- The top of the blade sits at the waterline so once \( x \) is larger than zero the blade begins leaving the water (see fig. 33 below)
- The rate of rotation of the blade is constant; there is no acceleration or deceleration.
- The drag coefficient, \( C_d \) equals 1.5.
- The velocity of the Current is constant across the blades surface

Fig. 33 Rotation of one blade through water

Fig. 34 Cross section of a blade
Expressing \( V_b \) and \( V_c \)

\[
\begin{align*}
\text{t} &= \text{time taken for one blade to rotate 360 degrees} \\
\text{Current} &= \text{velocity of the river} \\
x &= \text{angle at the centre of water wheel} \\
x_L &= \text{angle at the centre of the waterwheel as the blade leaves the water} \\
\text{Cd} &= \text{Drag Coefficient} \\
\rho &= \text{Density of Water}
\end{align*}
\]

In this early design model it was decided that the user would specify the time for one blade to rotate 360 degrees. This was later changed to be derived from the blades velocity. The angle that the blade would leave the water was also specified by the user regardless of the wheels dimensions. There is no DraftT value incorporated so the top of the blade is in line with the waterline, meaning the area decreases as \( x \) increases and so the power output was low.
Expressing Area, A

The dimensions of the wheel are decided by the user based on aesthetics, site conditions and minimising cost.

$x := 0..45$

\[ f := 0.3 \]

\[ d := 0.5 \]

\[ L := 1.3 \]

\[ y := L - d \]

\[ y = 0.8 \]

\[ a(x) := \frac{y}{\cos(x \text{ deg})} \]

\[ D(x) := L - a(x) \]

\[ A(x) := D(x) \cdot f \]

As the Force output is based on the Area this graph will give a low output as the area is decreasing with x. A more optimum design would have a constant area for as long as possible and then a quick drop in submerged area.
Expressing $V_r$

The lever arm changes as the blade leaves the water and the submerged area decreases

$$L_{arm}(x) := L - \left( \frac{D(x)}{2} \right)$$

$L_{arm}$ is the lever arm from the centre of the waterwheel

$$p := \frac{1}{3}$$

$p = \text{the optimum proportion of velocity the blade absorbs from the current. Constant.}$

$$V_b(x) := \text{Current} \cdot p$$

$V_b(x)$ = rotational velocity of the blade

$$V_c(x) := \text{Current} \cdot \cos(x \text{ deg})$$

$V_c(x)$ = component of the current in the blade's direction

$$V_r(x) := V_c(x) - V_b(x)$$

$V_r$ = relative velocity

As expected the relative velocity drops as the blade begins to leave the water
Finding the force on the blade

\[ F(x) := 0.5 \rho C_D (V(x))^2 A(x) \]

\( F(x) \) = Force on the blade

Finding the Moment at the waterwheel centre

\[ M(x) := F(x) \cdot L_{arm}(x) \]

\( M(x) \) = Moment at the waterwheel centre

Both graphs seem consistent with less Force and Moment being produced as the blade leaves the water.
Domestic electricity generation using waterwheels on moored barge

Expressing the Work Done and Power Absorbed

Work done in the 2nd half of travel = Work done in the first half of travel, so the Total Work Done on one paddle moving through the water= 2 * the integral of the Moment at the centre of the wheel

\[
\text{WorkDone} := 2 \int_{0}^{\pi L} M(x) \, dx
\]

\[
\text{WorkDone} = \text{work done on 1 blade whilst moving through the water}
\]

\[
N := \text{number of blades}
\]

\[
\text{TotalWorkDone} := N \cdot \text{WorkDone} \left( \frac{\pi}{180} \right)
\]

\[
\text{TotalWorkDone} = \text{total work done for the whole wheel. A conversion is needed to change degrees into radians}
\]

\[
\text{PowerAbsorbed1} := \frac{\text{TotalWorkDone}}{t}
\]

\[
\text{PowerAbsorbed1} = 120.584
\]

\[
\text{PowerAbsorbed} := \frac{\text{PowerAbsorbed1}}{1000}
\]

\[
\text{PowerAbsorbed} = 0.121
\]

PowerAbsorbed (kW) = total amount of power absorbed by the wheel during one movement through the water.
4.20 **Reviewing and Modification of MathCAD Model**

The basic model had a number of inaccurate features that could be improved; a copy of the improved worksheet can be seen over the next four pages. Improvements included:

- Introduction of a waterline and a DraftT value which then allows the designer to alter how low in the water the wheel is sitting.
- The exit angle, xL is now defined in terms of the draft and the diameter, L allowing a more integrated design.
- The change in submerged area is also linked to the draft and the model has been updated so the area is constant until the blade begins to leave the water.
- By rearranging time = distance/speed in terms of rotational distance/blade velocity a more accurate value for t is produced.

Fig. 36 Modified MathCAD design

**Expressing Vb and Vc**

Current1 := 2

DraftT := 0.4

L := 4.5

L= distance from centre of waterwheel to bottom of blade in metres

\[
xL := \left(\cos\left(\frac{\text{DraftT}}{L}\right)\right) \left(\frac{360}{2\pi}\right)
\]

xL = 84.9  

xL= angle at the centre of the waterwheel as the blade leaves the water

x := 0 .. xL  

x= angle at the centre of water wheel in degrees

\[\text{Cd} := 1.5\]

\[\rho := 1000\]

\[\rho=\text{Density of Water in kgm}^{-3}\]

A more realistic design is created by inputting the waterline and noting the Force created from x = 0 to x = xL.
Domestic electricity generation using waterwheels on moored barge

\[ f = \text{width of the blade in metres} \]
\[ d = \text{length of blade in metres} \]
\[ L = \text{distance from centre of waterwheel to bottom of blade in metres} \]
\[ y := L - d \]
\[ y = 1 \]
\[ \text{DraftT} = \text{Distance from centre of waterwheel to the water line when } x=0 \]

\[ a(x) := \frac{\text{DraftT}}{\cos(x \text{ deg})} \]
\[ a(x) = \text{diagonal distance from the centre of the waterwheel to the water line in metres} \]

\[ D(x) := \begin{cases} 
  d & \text{if } L - a(x) > d \\
  (L - a(x)) & \text{otherwise} 
\end{cases} \]
\[ D(x) = \text{depth of blade in water in metres} \]

\[ A(x) := D(x) \cdot f \]
\[ A(x) = \text{surface area of blade in water in } m^2 \]

The inclusion of a DraftT term allows the Area to be accurately calculated. The graph above shows the blade area being constant, as it moves through the water and then suddenly dropping as it leaves the water.
Expressing \( V_r \)

\( p = \text{proportion of speed blade takes up from river} \)

\( p = 0.33 \)

\[ t := \frac{2\pi L}{p \cdot \text{Current1}} \]

\( t = \text{time taken for one blade to do one revolution (seconds)} \)

Calculation of \( t \) is now linked to values of \( L \) and the Current speed

\[ t = 42.84 \]

\[ V_b := p \cdot \text{Current1} \]

\( V_b(x) = \text{rotational velocity of the blade in ms}^{-1} \)

\[ V_c(x) := \text{Current1} \cdot \cos(x \text{ deg}) \]

\( V_c(x) = \text{component of the Current in the blade's direction in ms}^{-1} \)

\[ V_r(x) := V_c(x) - V_b \]

\( V_r = \text{velocity of the blade relative to the speed of the Current in ms}^{-1} \)
Domestic electricity generation using waterwheels on moored barge

**Finding the force on the blade**

\[ F(x) := 0.5 \cdot \rho \cdot C_d \cdot (Vf(x))^2 \cdot A(x) \]

\[ F(x) = \text{Force on the blade in } \text{kgm}^2\text{s}^{-2} \text{ or Newtons} \]

**Finding the Moment at the waterwheel centre**

\[ L_{arm}(x) := (L) - \left( \frac{D(x)}{2} \right) \]

\[ L_{arm} \text{ is the lever arm from the centre of the waterwheel in metres} \]

\[ M(x) := F(x) \cdot L_{arm}(x) \]

\[ M(x) = \text{Moment at the waterwheel centre in } \text{kgm}^2\text{s}^{-2} \text{ or Nm} \]
Expressing the Work Done and Power Absorbed

Work done in the 2nd half of travel = Work done in the first half of travel, so the Total Work Done on one paddle moving through the water = 2\* the integral of the Moment at the centre of the wheel

\[
\text{WorkDone} := 2 \int_{0}^{xL} M(x) \, dx
\]

\[
N := 16
\]

\[
\text{TotalWorkDone} := N \cdot \text{WorkDone} \left( \frac{\pi}{180} \right)
\]

\[
\text{PowerAbsorbed1} := \frac{\text{TotalWorkDone}}{t}
\]

\[
\text{PowerAbsorbed1} = 2.477 \times 10^4
\]

\[
\text{PowerAbsorbed} := \frac{\text{PowerAbsorbed1}}{1000}
\]

\[
\text{PowerAbsorbed} = 24.771 \text{ kW}
\]

WorkDone = work done on 1 blade whilst moving through the water in Nm

N = number of blades

TotalWorkDone = total work done for the whole wheel in Nm

PowerAbsorbed1 = total amount of power absorbed by the wheel during one movement through the water in Nms^-1 or Watts

PowerAbsorbed = total amount of power absorbed by the wheel during one movement through the water in kNms^-1 or kWatts
4.21 **Further Modification**

It can be seen from the graph of $V_r(x)$ against $x$ that there comes a point when the velocity appears to be negative causing an upturn in the Force against $x$ graph (below). Considering $V_r = V_c - V_b$

$$= (\text{Current} \cdot \cos x) - (\text{Current} \cdot p)$$

$$= (\cos x) - p$$

So in that case when $\cos x = 1/3$ then $V_r$ will equal zero. This will occur regardless of the current at approximately 70.5 degrees (see graph below).

Fig. 37 Further Modification of MathCAD sheet

\[ p := 0.3 \]

\[ p = \text{proportion of speed blade takes up from river, proved in Appendix} \]

\[ V_b := p \cdot \text{Current 1} \quad V_b(x) = \text{rotational velocity of the blade in ms}^{-1} \]

\[ V_c(x) := \text{Current 1} \cdot \cos (x \cdot \text{deg}) \]

\[ V_r(x) := V_c(x) - V_b \]

$V_r$ = velocity of the blade relative to the speed of the Current in ms$^{-1}$

\[ F(x) := 0.5 \cdot \rho \cdot C_d \cdot (V_r(x))^2 \cdot A(x) \]

\[ F(x) \cdot 10^{-4} \]

\[ 2 \cdot 10^{-4} \]

\[ 0 \quad 20 \quad 40 \quad 60 \quad 80 \quad 100 \]

\[ x \]

\[ 0 \quad 0.5 \quad 1 \quad 1.5 \]

\[ V_r(x) \]
This can be physically reconciled by considering the blade’s path through the water (See Fig. 38). When the blade is vertical in the water the current velocity acts at right angles to the blade, generating the most force. As the blade rises the angle changes enabling more water to flow under the blade not passing on energy. At 70.5 degrees the current passes on no more energy to the blade managing to escape around the base and sides. Above 70.5 degrees it is suggested that the system may begin to lose energy as the blade push aside water above it to reach the surface.

Fig. 38 Diagram showing blades at 0 degrees and 70.5 degrees.

This implies that the blade need not be in the water past 70.5 degrees, which in turn decides the optimum Draft for the system;

\[
L := \text{5.5} \\
x1 := \text{70.5} \\
y := \text{2} \\
\text{DraftT} := \cos(x1 \text{deg}) \cdot y \\
\text{DraftT} = 0.668 \\
xL := \left(\cos\left(\frac{\text{DraftT}}{L}\right)\right) \left(\frac{360}{2\pi}\right) \\
xL = 83.028 \\
x := 0.. xL \\
x = \text{angle at the centre of water wheel in degrees}
\]
By adjusting the Vr equation all negative values of Vr are ignored, removing the upturn in the Force graph;

\[ V_b := \rho \cdot \text{Current1} \]
\[ V_b(x) = \text{rotational velocity of the blade in } \text{ms}^{-1} \]

\[ V_c(x) := \text{Current1} \cdot \cos(x \text{ deg}) \]
\[ V_c(30) = 1.732 \]

\[ V_r(x) := \begin{cases} 
(V_c(x) - V_b) & \text{if } (V_c(x) - V_b) > 0 \\
0 & \text{otherwise} 
\end{cases} \]

\[ F(x) := 0.5 \cdot \rho \cdot C_d \cdot (V_r(x))^2 \cdot A(x) \]
\[ F(x) = \text{Force on the blade in } \text{kgms}^{-2} \text{ or Newtons} \]

The only assumptions still in the final worksheet were:

- The drag coefficient, Cd equals 1.5.
- The rate of rotation stays constant, there is no acceleration or deceleration of the blades.
- The velocity of the Current acts constantly across the blades surface.
- The forces experienced by the blade are simplified not considering the effects of turbulence, currents or loss of energy through splashing.
- Blades are flat, rectangular blocks.
- Constant \( p \) is kept at 1/3 by later generator selection and design.
4.2 Variables Testing

By altering the input variables individually by 50% and noting the corresponding Power Absorbed the most influential variables can be found, leading to a more complete understanding of how they influence power output.

4.31 Initial Values- number of paddles, N =7

- Current= 2 ms^-1
- DraftT= 0.8m
- Width of blade, f= 0.3m
- Depth of blade, d= 0.5m
- Length from axis to base of blade, L= 1.3m
- Lever arm, y= 0.8m
- Constant, p= 0.33
- Angle blade leaves the water, xL= 52.02 deg

This gives an initial Power Output of 0.12 kW

4.32 Test 1.0- Confirmation of p=0.33 value:

i) Values of p from 0-1.0 were plotted against power output and the maximum was found to lie between 0.2 and 0.4

![Graph showing power output against p]

\[
p := \begin{pmatrix} 0.112 \\ 0.173 \\ 0.193 \\ 0.183 \\ 0.15 \\ 0.107 \\ 0.062 \\ 0.027 \\ 9.896 \times 10^{-3} \\ 0.022 \end{pmatrix}
\]

\[
\text{Power} := \begin{pmatrix} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \\ 1.0 \end{pmatrix}
\]
ii) Values of p from 0.20-0.39 were plotted against Power Output values (kW) with the maximum occurring at 0.3 confirming the proof work in the Technical Background chapter.

![Graph showing the relationship between p and Power1.]

\[
\begin{pmatrix}
0.20 & 0.21 & 0.22 & 0.23 & 0.24 \\
0.25 & 0.26 & 0.27 & 0.28 & 0.29 \\
0.30 & 0.31 & 0.32 & 0.33 & 0.34 \\
0.35 & 0.36 & 0.37 & 0.38 & 0.39 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
0.173 \\
0.177 \\
0.18 \\
0.183 \\
0.185 \\
0.187 \\
0.189 \\
0.191 \\
0.192 \\
0.192 \\
0.192 \\
0.193 \\
0.193 \\
0.193 \\
0.192 \\
0.191 \\
0.189 \\
0.188 \\
0.186 \\
0.184 \\
\end{pmatrix}
\]

**Test 1.1- Change of blade area:**

i) Increase in blade width, f by 50%. Power Output noted.

ii) Increase in blade depth, d by 50% which led to increases in length, L and leaving angle of blade, xL. Power Output noted.

**Test 1.2- Change in number of blades:**

i) Increase in number of blades by 57%. Power Output noted.

**Test 1.3- Change in lever arm:**

i) Increase in lever arm, y by 50% leading to an increase in length, L and leaving angle of blade, xL. Power Output noted.

**Test 1.4- Change in current speed:**

i) Current speed increased by 50%. Power Output noted.

**Test 1.5- Change in DraftT**

i) Increase in DraftT, causing a decrease in leaving angle of blade, xL. Power Output noted.

ii) Decrease in DraftT, causing an increase in leaving angle of blade, xL. Power Output noted.
Fig. 39 Results from Tests 1.1-1.5. Blue variables have been changed; yellow shows other variables reacting to that change

<table>
<thead>
<tr>
<th>Test</th>
<th>N</th>
<th>Current (ms^-1)</th>
<th>Draft T (m)</th>
<th>f (m)</th>
<th>d (m)</th>
<th>L (m)</th>
<th>y (m)</th>
<th>p</th>
<th>xL</th>
<th>Power Output (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Values</td>
<td>7</td>
<td>2</td>
<td>0.8</td>
<td>0.3</td>
<td>0.5</td>
<td>1.3</td>
<td>0.8</td>
<td>0.3</td>
<td>52.02</td>
<td>0.12</td>
</tr>
<tr>
<td>T1.1</td>
<td>i)</td>
<td>7</td>
<td>0.45</td>
<td>0.5</td>
<td>1.3</td>
<td>0.8</td>
<td>0.3</td>
<td>52.02</td>
<td>0.187</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>0.75</td>
<td>1.55</td>
<td>0.8</td>
<td>0.3</td>
<td>58.9</td>
<td>0.19</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ii)</td>
<td>7</td>
<td>2</td>
<td>0.8</td>
<td>0.3</td>
<td>0.5</td>
<td>1.3</td>
<td>0.8</td>
<td>0.3</td>
<td>52.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>0.12</td>
<td>1.2</td>
<td>1.3</td>
<td>0.8</td>
<td>0.3</td>
<td>52.02</td>
<td>0.421</td>
<td>0.158</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>0.4</td>
<td>0.5</td>
<td>1.3</td>
<td>0.8</td>
<td>0.3</td>
<td>72.08</td>
<td>0.158</td>
<td>0.158</td>
</tr>
</tbody>
</table>

From the table above can be seen the increases in the variables and the corresponding increases in Power Output. The most influential variable for a stream wheel is the Current velocity shown by Test 1.4 as with a 50% increase it produces a 350% increase in Power Output. This is consistent with the relative velocity, Vr value being to the power of 2 in the Force equation.

The second most influential variable, which can be controlled by the designer is the number of blades. However the MathCAD model does not take into account that too many blades will lead to the water not being able to flow freely around the blade and so less force being generated on each blade. It was felt between 12 and 16 blades was optimum.

The tests show that an increase in the lever arm, y is more influential than either an increase in the depth or width of the blade (which produce very similar Power Output increases) and any change in draft is surprisingly uninfluential.
4.4 **Estimation of Typical Domestic Energy Requirements**

There are no published figures for a typical 3-bedroom house’s energy requirements so in order to estimate them a list of household appliances was drawn up, their power ratings and duration of usage estimated. Obviously not all of these appliances will be running all day every day, so the duration of usage (in a month) is estimated and multiplied by the rating to get the power usage in kWhr. This is then divided by the number of hours in a month to get an overall power requirement in kW.

Standard appliances have been used throughout as have some of the findings of the Energy Consumption in the UK Report (41) which shows statistically appliances that are widely owned. From the graph below the household appliances considered “typical” included- VCR, washing machine, microwave, fridge freezer, refrigerator.

![Percentage of households that own domestic appliances](image)

Fig. 40 Graph showing the percentage of household appliances owned in the UK, 1970-2000 (41)

From page 25 of the report it was noted that 59% of UK households have two or more colour televisions, 45% of UK houses own a home computer and that energy consumption for lighting had risen over the last decade due to multiple lights and table lamps (41). According to more recent DTI (42) data electric heating is the second most common form of heating, far behind gas so it is fairly realistic for the house in this dissertation to be heated using electricity.
Fig. 41 Table calculating the typical energy requirements of a three bedroom, four person family based on figures from (43).

<table>
<thead>
<tr>
<th>Appliance</th>
<th>Quantity</th>
<th>Power rating in W</th>
<th>Average hours per month</th>
<th>Energy used (kWhr) per month</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blender</td>
<td>1</td>
<td>350</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Coffee Maker</td>
<td>1</td>
<td>900</td>
<td>12</td>
<td>11</td>
</tr>
<tr>
<td>Electric Kettle</td>
<td>1</td>
<td>1500</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>Refrigerator freezer</td>
<td>1</td>
<td>500</td>
<td>300</td>
<td>150</td>
</tr>
<tr>
<td>Microwave Oven (0.5 ft)</td>
<td>1</td>
<td>900</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>Range and oven</td>
<td>1</td>
<td>3500</td>
<td>25</td>
<td>90</td>
</tr>
<tr>
<td>Toaster</td>
<td>1</td>
<td>1200</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Washing machine (33 loads/month)</td>
<td>1</td>
<td>500</td>
<td>26</td>
<td>13</td>
</tr>
<tr>
<td>Electric water heaters</td>
<td>1</td>
<td>3800</td>
<td>140</td>
<td>532</td>
</tr>
<tr>
<td>Family of four</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Iron</td>
<td>1</td>
<td>1000</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Water pump (1.2 hp)</td>
<td>1</td>
<td>1000</td>
<td>44</td>
<td>44</td>
</tr>
<tr>
<td>Electric Heating</td>
<td>1</td>
<td>1000</td>
<td>250</td>
<td>250</td>
</tr>
<tr>
<td>Hair Dryer</td>
<td>1</td>
<td>1000</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>60 W bulbs</td>
<td>15</td>
<td>900</td>
<td>120</td>
<td>108</td>
</tr>
<tr>
<td>Portable electric heater</td>
<td>1</td>
<td>1000</td>
<td>350</td>
<td>350</td>
</tr>
<tr>
<td>Telephone</td>
<td>1</td>
<td>6</td>
<td>720</td>
<td>4</td>
</tr>
</tbody>
</table>
## Domestic electricity generation using waterwheels on moored barge

<table>
<thead>
<tr>
<th>Appliance</th>
<th>Quantity</th>
<th>Power Rating in Watts</th>
<th>Average hours per month</th>
<th>Energy used (kWhr) per month</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home computer</td>
<td>1</td>
<td>250</td>
<td>240</td>
<td>60</td>
</tr>
<tr>
<td>Radio</td>
<td>1</td>
<td>5</td>
<td>120</td>
<td>1</td>
</tr>
<tr>
<td>Stereo</td>
<td>1</td>
<td>120</td>
<td>120</td>
<td>14</td>
</tr>
<tr>
<td>Colour Television</td>
<td>1</td>
<td>100</td>
<td>125</td>
<td>13</td>
</tr>
<tr>
<td>Video Cassette Recorder</td>
<td>1</td>
<td>40</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td><strong>Total Energy Usage</strong></td>
<td></td>
<td></td>
<td></td>
<td><strong>1694 kWhr</strong></td>
</tr>
<tr>
<td><strong>Number of hours per month</strong></td>
<td></td>
<td></td>
<td></td>
<td><strong>720 hrs</strong></td>
</tr>
<tr>
<td><strong>Total Power Rating Required</strong></td>
<td></td>
<td></td>
<td></td>
<td><strong>2.35 kW</strong></td>
</tr>
</tbody>
</table>

The average power rating is 2.35 kW but obviously there will be periods where there is a higher consumption e.g. morning breakfast and evenings, winter time. For their electrical system the Pedley Wheel Trust estimates efficiencies in the region of generator- 81% and gearbox- 97% (49). Using these figures the overall power output will be 7.857 kW. By choosing 10 kW as the design output for the wheel enough energy can be insured and excess power may be stored in a battery or sold back to the national grid.
4.3 Final Waterwheel Specification

The final waterwheel output is 10.5kW of power with specifications of:

- \( L = 3.5 \text{m} \)
- \( y = 1 \text{m} \)
- \( d = 2.5 \text{m} \)
- \( f = 3 \text{m} \)
- \( \text{DraftT} = 0.42 \text{m} \)
- \( N = 12 \)

4.41 Rotational speed check

One of the "main disadvantages of water wheels for electricity generation is the slow shaft speed" (24) as AC generation requires a high rotational speed. Ideally at the most a 1:100 gear ratio will be used needing a rotational speed of at least 6 rpm from the wheel. Using \( \text{Rotational speed of the wheel} = (9 \times \text{Current}) \div \text{Diameter} \) (6) the rotational speed of the final waterwheel was calculated as being 2.75 rpm. Chapters 5 and 6 detail a barge and catamaran design to support the wheel and also increase this rotational speed.
5  **Barge Design using Excel**

Following the calculation of a low rotational speed of waterwheel one possible solution was suggested- a barge with two waterwheels either side. A breakdown of the tasks involved in the barge design is shown below:

![Design process for barge](Fig. 42 Design process for barge)
5.1 **Basic Excel Model**

Using the basic naval architecture given in the Technical Background section the basic barge design spreadsheet over pages 62-64 was generated (see enclosed CD for spreadsheet). The triangular barge ends pictured below prevent waves and turbulence at the barge ends. For this small scale design it was felt that the metacentric height, GM should be at least 30cm. Although drawn here the trash rack is not included in the spreadsheet.

![Cross section showing barge with two waterwheels attached at either side](image1)

**Fig. 43** Cross section showing barge with two waterwheels attached at either side

![Plan of barge showing approximate layout of generator, gearbox and watertanks](image2)

**Fig. 44** Plan of barge showing approximate layout of generator, gearbox and watertanks
5.2 Waterwheel Re-design
The decision to have two 5.32 kW waterwheels producing the same 10.64 kW total power output as one wheel would have, leads to a lesser diameter for each wheel, increasing the rotational speed of the wheels, lessening the gear ratio.

Working in reverse from the optimisation of 4.32 now it becomes a matter of which dimensions can be lessened, whilst still keeping power output as high as possible. In changing the diameter, \( L \) the depth of the blades, \( d \) and the lever arm, \( y \) will lessen. Tests 1.1 and 1.3 showed that the depth of the blade is more influential in power output terms, so is kept as high as possible.

The final dimensions for each wheel are:
\[
\begin{align*}
L &= 1.5m \\
y &= 0.4m \\
d &= 1.1m \\
f &= 3.5m \\
\text{DraftT} &= 0.169m \\
N &= 12
\end{align*}
\]

The rotational speed for each wheel is now 6rpm allowing a gear ratio of 1:100.

It is assumed that both wheels will experience the same velocity of current and so rotate at the same speed.
5.3 Generator, Gearbox and Watertank selection

5.31 Gearbox selection

A disadvantage often sited about waterwheels is their slow rotational speed. In the industrial revolution when they were often used to mill or lift water this posed no problem but to generate AC electricity a large gearing ratio is needed.

Waterwheel installers have sought to overcome this in a number of ways- using tractor gearboxes (6), modified car alternators (47), or even washing machine motors (48). Some groups advocate using integral geared motors, describing them as being available worldwide and relatively cheap (49) whereas others claim using them as a generator is expensive (6).

The system shown below uses a specially designed Hydro-alternator that is ideal for low rotational speeds and is self-adjusting, and so useful for variable flows. The Canadian manufacturer supplies remote fishing cabins that generate their own electricity for domestic use and fishing boat battery recharging (50). Notice by building a sloped offshoot stream to the main river the velocity can be increased.

Fig. 45 Diagram showing Hydro-alternator wired to a waterwheel, Canada (50)

For the purposes of this dissertation a gearbox has been chosen to give a fairly accurate idea of the typical weight and dimensions involved, to more accurately calculate the downwards force on the barge and the space required. For a real-life design a trained mechanical engineer would be consulted.

Following advice from Dr. Davies of Heriot Watt University a type of reduction gearbox was chosen from suppliers, Boston Gear’s (www.bostongear.com) non-flanged
reducer 600 series range. Gearbox 623A-100 has been chosen with a ratio of 1:100 and weight of 36 lbs ($\approx$16.3 kg) (51) with the overall dimensions in Fig…

![Overall dimensions of gearbox from (51)](image)

**Generator Selection**

As most houses require AC electricity it was decided to pick a Permanent Magnet Alternator that produces AC current. Other generation and storage systems could be selected but this is just a preliminary selection to give some indication of the weight and dimensions. After much searching a 2.5 kW Alternator was selected and its weight and overall dimensions multiplied by 1.5 to roughly equal a 5 kW Alternator, giving weight $\approx$ 60 kg (52)

![Overall dimensions of generator (52)](image)

**Watertank Design**

In barge or catamaran design it is necessary to exactly balance the downwards force of the barge and its contents, with the upwards thrust of the volume of water displaced. To allow for future design changes that lead to an increase or decrease in weight (such as new, heavier gearbox or more blades on waterwheels, lighter generator etc.) two water tanks are included in this design. If further load is added later the equivalent weight of water can be removed from the tanks, keeping the vessel at the same draft depth.
5.34 Iterative barge design

There are two main components to designing a barge; on one hand the force downwards has to equal the force upwards and the stability criteria have to be met. The downwards force of the barge and its contents is balanced by the volume of water the barge is displacing. In that case it follows that a greater underwater volume will lead to a greater load supported.

However the metacentric height, GM is calculated using \( GM = VCB + BM - VCG \) where VCB is the Vertical Centre of Buoyancy, BM is the distance between the centre of buoyancy and the metacentre and VCG is the Vertical Centre of Gravity. This can be rewritten:

\[
GM = (Draft \div 2) + (2nd\text{MomentofArea},Ixx \div Underwatervolume) - ((\sum Mass \times \text{DistToCentreofGravity}) \div \sum Mass)
\]

The draft value is given from the MathCAD sheet and ideally is not altered. So to maximise the GM value BM has to be at a maximum and VCG at a minimum.

\[
BM = (bd^3 / 12) \div (l \times b \times (d - Draft))
\]

It is difficult to maximise BM as increasing the top line containing breadth and depth will lead to a proportional increase in the bottom line, as it also contains breadth and depth.

Working with the Excel spreadsheet it seems as though maximising the length and minimising the depth and breadth slowly increases the metacentric height, however despite several design changes, such as lighter material for the wheel (wood instead of steel), a low 4 kN of watertank weight and the inclusion of triangular barge ends, contributing to the underwater volume the metacentric height still failed to rise above zero. Over the following three pages a copy of the Excel file can be seen with the two forces balancing but the metacentric height still being too low.

It was felt that this indicated the unsuitability of a barge to support two waterwheels, especially two wheels that had to be set low into the flow to generate high enough outputs. It was therefore decided to modify the Excel file to investigate a catamaran layout (Chapter 6). This would not need to be as rigorously analysed for stability as catamarans are inherently more stable, having two hulls rather than one.
### Barge Design Spreadsheet

**Downwards Force**

#### Waterwheel

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Density of blade (kg/m^3)</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of blades, N</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Draft required, T (m)</td>
<td>0.169</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length of blade, d (m)</td>
<td>1.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depth of blade, b (m)</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Width of blade, f (m)</td>
<td>3.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total volume of 1 blade (m^3)</td>
<td>0.385</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total volume of all blades (m^3)</td>
<td>4.62</td>
<td>Density of blade (kg/m^3)</td>
<td>500</td>
</tr>
<tr>
<td>Radius of paddle, r (m)</td>
<td>0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length of 1 paddle, y (m)</td>
<td>0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total volume of all paddles (m^3)</td>
<td>0.0377</td>
<td>Density of paddle (kg/m^3)</td>
<td>500</td>
</tr>
<tr>
<td>Diameter of axis pole (m)</td>
<td>0.2</td>
<td>Length of axis pole (m)</td>
<td>4.5</td>
</tr>
<tr>
<td>Mass of axis pole (kg)</td>
<td>70.6858</td>
<td>Density of axis pole (kg/m^3)</td>
<td>500</td>
</tr>
<tr>
<td>Total mass of blades (kg)</td>
<td>2310</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total mass of paddles (kg)</td>
<td>18.8496</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total downwards acting Force from both wheels (kN)</td>
<td>47.0789</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance from base of barge to CoG for waterwheel (m)</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Gearbox

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Mass of two gearboxes (kg)</th>
<th>32.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of gearbox (see report for details) (kg)</td>
<td>16.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total downwards acting Force (kN)</td>
<td>0.31981</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depth of gearbox (m)</td>
<td>0.161</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance from base of barge to Centre of Gravity (m)</td>
<td>0.0805</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Generator

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Mass of two generators (kg)</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of generator (see report for details) (kg)</td>
<td>60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total downwards acting Force (kN)</td>
<td>2.3544</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depth of generator (m)</td>
<td>0.408</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance from base of barge to Centre of Gravity (m)</td>
<td>0.204</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Main body of Barge

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, lb (m)</td>
<td>8</td>
</tr>
<tr>
<td>Breadth, Bb (m)</td>
<td>2</td>
</tr>
<tr>
<td>Depth, Db (m)</td>
<td>1</td>
</tr>
<tr>
<td>Thickness (m)</td>
<td>0.2</td>
</tr>
<tr>
<td>Volume of base (m^3)</td>
<td>3.2</td>
</tr>
<tr>
<td>Mass of base (kg)</td>
<td>1600</td>
</tr>
<tr>
<td>Distance from base of barge to Centre of Gravity (m)</td>
<td>0</td>
</tr>
<tr>
<td>Volume of sides A (m^3)</td>
<td>0.8</td>
</tr>
<tr>
<td>Volume of Sides B (m^3)</td>
<td>3.2</td>
</tr>
<tr>
<td>Mass of sides (kg)</td>
<td>2000</td>
</tr>
<tr>
<td>Distance from base of barge to Centre of Gravity (m)</td>
<td>0.5</td>
</tr>
</tbody>
</table>

### Triangular Barge Ends

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of side C (m)</td>
<td>1.6</td>
</tr>
<tr>
<td>Volume of sides C (m^3)</td>
<td>1.28</td>
</tr>
<tr>
<td>Mass of Sides C (kg)</td>
<td>2048</td>
</tr>
</tbody>
</table>

| Volume from base of barge to centre of gravity (m) | 0.5 |
| Distance q (m)                              | 1.249 |
| Volume of base of triangular barge ends (m^3) | 0.1249 |
| Mass of base of triangular barge ends (kg)    | 62.45 |
| Downwards force (kN)                       | 1.23  |
Domestic electricity generation using waterwheels on moored barge

**Barge Design Spreadsheet**  

**Watertanks**

- Length of Side F (m) 0.75
- Length of Side E (m) 0.4
- Depth of watertank, Dw (m) 1
- Level of water in watertank, dw (m) 0.3
- Mass (kg) of watertanks with water at level dw (kg) 180
- Distance from base of barge to Centre of Gravity (m) 0.15

- Downwards force exerted by both watertanks (kN) 1.7658

**Total Downwards force (kN)** 108.151

**Upwards Force**

- Volume water displaced by barge (m^3) 11.049

**Total Upwards Force (kN)** 108.39

**Barge Design Spreadsheet**  

**VCB (m)** 0.0845

**VCG**

- Base and triangular base Moment (kgm) 0
- Sides Moment (kgm) 1000
- Watertank Moment (kgm) 27
- Gearbox Moment (kgm) 2.6243
- Generator Moment (kgm) 24.48
- Waterwheel and axis pole Moment (kgm) 2399.53591
- Side C Moment (kgm) 1024
- Side D Moment (kgm) 0

- Sum of Moments (kgm) 4477.639691

**VCG (m)** 0.530363567

**I waterplane about Centre line (m^4)** 0.166666667

**BM (m)** 0.015084354

**Metacentric Height, GM (m)** (should be less than 0.30m) -0.430779213
6. Catamaran Design

Following the unsuccessful barge design a catamaran design was suggested. The original problem of a slow turning wheel was now solved by using the sides of the catamaran to restrict flow towards the wheel, increasing the speed. Test 1.4 earlier showed that an increase in Current of 50% could lead to an increase in power output of 350% allowing the wheel diameter to be lessened, increasing the rotational speed of the wheel.

Taking the modified MathCAD output the Excel model was altered to design a catamaran (see enclosed CD for catamaran file) and an iterative process was begun to balance forces in both directions.

Fig. 49 Diagram showing design process followed for catamaran
6.1 **Catamaran layout and Re-design of Waterwheel**

6.11 **Catamaran Layout**

The catamaran design involves a wheel in between two hulls containing the gearbox, generator and water tanks as symmetrically as possible (See Figs 50 and 51 below). Although shown here neither the trash rack nor stability bars are considered in the Excel file, being fairly lightweight.

**Fig. 50 Cross section of catamaran in direction of flow**

**Fig. 51 Plan of catamaran**
6.12 **New waterwheel design**

To evaluate the increase in Current caused by a restricted opening area to the wheel firstly the flow in the river has be inputted. As actual data is not available it is estimated that the channel is 4m wide and the distance from the waterline to the river bed is 3m. If site investigations showed otherwise these figures can be changed manually in the MathCAD program (see attached CD for program). From this a flow rate of 24 m$^3$s$^{-3}$ was calculated. The restricted area of the flow before the wheel is equal to the multiplication of the catamaran depth (kept fairly deep to ensure high velocities and water flowing into wheel rather than underneath catamaran) and the distance between the two hulls, set at 0.5m more than the width of the blades. In this example using this arrangement increases the Current from 2 m$s^{-1}$ to 3.5 m$s^{-1}$

This increased speed allows for smaller wheel dimensions leading to an increase in the rotational speed of the wheel. The final wheel dimensions are:

- $L = 1.25$m
- $y = 0.5$m
- $d = 0.75$m
- $f = 1.75$m
- $\text{DraftT} = 0.211$m
- $N = 12$

This gives an output of 10.76kW turning at a speed of 12.6 rpm.

### 6.2 Generator and Gearbox Selection

Although in a real life application a different gearbox and generator would be selected the same pair are being kept here. The selection in this dissertation would probably not be actually used in reality; here it is an indicator of size and weight. Instead of a 1:100 gear ratio 1:50 could be used here, and again two 5kW generators would be incorporated.
6.3 **Catamaran Design**

The Excel sheet for the barge was altered to include the two hulls of the catamaran (see enclosed CD for program). The new dimensions of the wheel were then added and the barge dimensions and water level in the tank were altered to balance the downwards and upwards forces. As catamarans are more stable than barges it was considered unnecessary to calculate the metacentric height of the system. The following 72-73 pages show the Catamaran Excel sheet.

The significant drop in the waterwheel weight means that the wheel can now be manufactured out of steel (coated against corrosion) with a density of 7700 kgm$^{-3}$ rather than the wood used for the barge design with a density of 500 kgm$^{-3}$. The catamaran is still made from wood, however.

The increased displacement volume of two hulls allows for 34 kN of water to be stored on the catamaran allowing large weight changes to be made in the future. This could help with future maintenance e.g. several people standing on the catamaran to clear the trash rack or bringing on heavy tools to repair or replace parts. The dimensions of the catamaran and water tanks have been kept uniform to ease construction, although a scale would be needed on the water tank for 1.78m of water to be measured out easily.
### Catamaran Design Spreadsheet

**Downwards Force**

**Waterwheel**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of blades, N</td>
<td>12</td>
</tr>
<tr>
<td>Draft required, T (m)</td>
<td>0.211</td>
</tr>
<tr>
<td>Length of blade, d (m)</td>
<td>0.75</td>
</tr>
<tr>
<td>Depth of blade, b (m)</td>
<td>0.1</td>
</tr>
<tr>
<td>Width of blade, f (m)</td>
<td>1.75</td>
</tr>
<tr>
<td>Total volume of 1 blade (m^3)</td>
<td>0.13125</td>
</tr>
<tr>
<td>Total volume of all blades (m^3)</td>
<td>1.575</td>
</tr>
<tr>
<td>Radius of paddle, r (m)</td>
<td>0.05</td>
</tr>
<tr>
<td>Length of 1 paddle,y (m)</td>
<td>0.5</td>
</tr>
<tr>
<td>Total volume of all paddles (m^3)</td>
<td>0.04712389</td>
</tr>
<tr>
<td>Diameter of axis pole (m)</td>
<td>0.2</td>
</tr>
<tr>
<td>Mass of axis pole (kg)</td>
<td>665.2322444</td>
</tr>
<tr>
<td>Total mass of blades (kg)</td>
<td>12127.5</td>
</tr>
<tr>
<td>Total mass of paddles (kg)</td>
<td>362.8593515</td>
</tr>
<tr>
<td>Total downwards acting Force from wheel (kN)</td>
<td>129.0563006</td>
</tr>
</tbody>
</table>

**Gearbox**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of gearbox (see report for details) (kg)</td>
<td>16.3</td>
</tr>
<tr>
<td>Total downwards acting Force (kN)</td>
<td>0.319806</td>
</tr>
<tr>
<td>Depth of gearbox (m)</td>
<td>0.161</td>
</tr>
</tbody>
</table>

**Generator**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of generator (see report for details) (kg)</td>
<td>60</td>
</tr>
<tr>
<td>Total downwards acting Force(kN)</td>
<td>2.3544</td>
</tr>
<tr>
<td>Depth of generator (m)</td>
<td>0.408</td>
</tr>
</tbody>
</table>

---

Red= input values
Domestic electricity generation using waterwheels on moored barge

**Catamaran Design Spreadsheet**

**Main body of Catamaran**

| Length, lb (m) | 2.5 |
| Breadth, Bb (m) | 1.5 |
| Depth, Db (m) | 3 |
| Thickness (m) | 0.2 |
| Volume of base (m³) | 0.75 |
| Mass of base (kg) | 375 |

| Volume of sides A (m³) | 1.8 |
| Volume of Sides B (m³) | 3 |
| Mass of sides (kg) | 2400 |

**Triangular Barge Ends**

| Length of side C (m) | 1.6 |
| Volume of sides C (m³) | 3.84 |
| Mass of Sides C (kg) | 1440 |
| Distance q (m) | 1.413329403 |
| Volume of base of triangular barge ends (m³) | 0.105999705 |
| Mass of base of triangular barge ends (kg) | 52.99985259 |

**Downwards force**

| Density (kgm⁻³) | 500 |
| Downwards force (kN) | 3.68 |
| Density (kgm⁻³) | 500 |
| Downwards force (kN) | 23.5 |
| Density (kgm⁻³) | 1000 |
| Downwards force (kN) | 1.04 |

**Catamaran Design Spreadsheet**

**Watertanks**

| Length of Side F (m) | 1 |
| Length of Side E (m) | 1 |
| Depth of watertank, Dw (m) | 3 |
| Level of water in watertank, dw (m) | 1.78 |
| Mass (kg) of watertanks with water at level dw (kg) | 3560 |

| Density of water (kgm⁻³) | 1000 |

| Downwards force exerted by all watertanks(kN) | 34.9236 |

**Total Downwards force (kN)**

| 286.3557208 |

**Upwards Force**

| Volume water displaced by catamaran (m³) | 29.16945375 |

**Total Upwards Force (kN)**

| 286.1523413 |
7 Conclusion

7.10 The waterwheel has played an important part in energy production over the last 2000 years, reaching its peak in usage in the early 19th century, but in the end losing out to steam and motor power. The waterwheel is today experiencing renewed interest as engineers, scientists and now energy suppliers (due to the Renewables Obligation) look for new renewable energies to combat climate change.

7.11 However progress is hampered by a lack of government support, high gearing ratios, little recent research and a perception amongst engineers, scientists and the public that waterwheels are old fashioned and irrelevant.

7.12 Waterwheel progress is most apparent on the internet with many sites written by home enthusiasts and small charities. This dissertation includes three case studies as a homage to this group and an example of how interested and keen the public are on interactive renewable energy.

7.13 Scotland has been chosen for this dissertation because of its plentiful supply of small rivers and streams, the high yearly rainfall and a rural population who could benefit from decentralised power. After briefly evaluating different types of wheel an undershot wheel was chosen, as it utilised the lowest heads. From this group the stream wheel was picked for this project as the site had zero head, and a fairly fast flow. Additionally stream wheels are simple to manufacture and considered “fish friendly”. Experiments currently running at Berlin Technical University will be able to shed new light onto their flow and power output capabilities.

7.14 A mathematical model of the flow of a stream wheel and its corresponding power output was created. The program was then used to estimate the dimensions for a typical domestic house’s energy requirements. The slow rotational speed of this design and the consequent large gearing ratio led to an investigation in fixing the wheel to a barge, and later a catamaran.
7.15 A basic barge layout complete with water tanks, generators and gearboxes was modelled in Excel and from the output it was concluded that the stability of a barge with two waterwheels either side was not satisfactory. So the spreadsheet was altered to describe a catamaran design with two hulls either side of a waterwheel. The benefit of this was that the entrance area of the water between the hulls was restricted, leading to a higher Current and so a smaller wheel for the same power output. Further advantages to this system included a high rotational speed, low gearing ratios, and a large amount of on board stored water, which allows later design alterations.

7.16 In conclusion it can be said that a waterwheel is a viable, aesthetically pleasing option for domestic energy generation that would benefit from further investigation.
8 Suggestions for future work

8.10 Being so previously unresearched waterwheels now present a large variety of potential projects. Fundamentally there has been little work into the characteristics of the flow of waterwheels. Understanding exactly how energy is transferred, how the blade interacts with the water and how these characteristics can be optimised is crucial to the advancement of all wheels especially the stream wheel, and the stream wheel in a rectangular channel.

8.11 From this predominately experimental and case study based work designs can be generated for the blade shape and size (with the drag coefficient being researched experimentally), optimum number of blades, and external works. An evaluation into the relative merits of fixed waterwheels perhaps sited in their own diversion channel or moored waterwheels on barges and catamarans is needed. Ecological studies are needed into the effects of waterwheels as is the simulation and minimisation of the impact noise so as to prevent wheels becoming unpopular with their neighbours.

8.12 In the wider context an evaluation is needed of the low head (< 25 kW) sites in the UK with more accurate estimates of the potential of microhydropower. A lack of knowledge of previous waterwheel installations also hampers progress so an in-depth investigation into case studies worldwide would enable a type of guide to be drawn up detailing successful practice. When combined with experimental and scientific theory this guide could slowly replace the wealth of intuitive knowledge lost at the end of the industrial revolution. Further afield the application of waterwheels to developing world sites is virtually unknown.

8.13 Any form of renewable energy will rely on generators, gearboxes, alternators or batteries to gain and store power and waterwheels are no exception. Further research is needed into gearing methods that are both efficient and easy to construct, and relatively inexpensive.
References


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   http://www.saveenergy.co.uk/renewables/explained/

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http://www.energy.soton.ac.uk/ then click on Current Research Work followed by Micro Hydropower- More about the Programme

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Appendix A

Project Planning Documents

Project Gantt chart for Term 1

Task A- Background Reading
Task B- Understand Fluid Mechanics involved in waterwheels
Task C- Research and make contact with industrial contacts
Task D- Produce preliminary waterwheel design
Trip to Barcelona
Task E- Type preliminary report. Send report to J.Wolfram and any industrial contacts
Project Planning Documents

Project PERT Chart for Term 1
# General Risk Assessment Form

<table>
<thead>
<tr>
<th>Project Title:</th>
<th>Dissertation - Domestic electricity generation using waterwheels on moored barge.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name:</td>
<td>Zoë Jones</td>
</tr>
<tr>
<td>Date:</td>
<td>June 2004 - May 2005</td>
</tr>
</tbody>
</table>

**Work Activities:**
1. Analysis of waterwheel carried out using computer and computer software, MathCAD.
2. Research and background reading carried out using computer and Internet and library resources.

<table>
<thead>
<tr>
<th>Hazards</th>
<th>Who might be harmed?</th>
<th>Existing controls</th>
<th>Is risk adequately controlled?</th>
<th>Further action to control</th>
</tr>
</thead>
<tbody>
<tr>
<td>(List significant hazards)</td>
<td>(List groups of people at significant risk)</td>
<td>(List existing controls)</td>
<td>(Y/N)</td>
<td>(List the risks which are not adequately controlled and the action necessary)</td>
</tr>
<tr>
<td>1. Repetitive Strain Injury (RSI) due to typing and using computer mouse for prolonged periods.</td>
<td>Student</td>
<td>Health and Safety (Display Screen Equipment) Regulations 1992. Health advice to take regular breaks</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>2. Eyestrain due to prolonged periods sitting at a flickering screen.</td>
<td>Student</td>
<td>Regular breaks and eye exercises</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>3. Sore joints and back due to hunched posture at a computer or desk</td>
<td>Student</td>
<td>Adjustable seats</td>
<td>N</td>
<td>Stretching and strengthening exercise to improve overall posture</td>
</tr>
<tr>
<td>4. Eyestrain due to reading in poor lighting</td>
<td>Student</td>
<td>Rooms allow natural light in and all have electric light</td>
<td>Y</td>
<td></td>
</tr>
</tbody>
</table>
Appendix B

Proof of “p” by J. Wolfram

\[ \text{Power} = \text{Force} \times \text{Blade velocity} \]

\[ \text{Power} = (0.5 \times \rho \times Cd \times A \times V_r^2) \times V_b \]

\[ V_r = V_c - V_b \]

\[ \text{Power} = (0.5 \times \rho \times Cd \times A \times (V_c - V_b)^2) \times V_b \]

\[ \text{Power} = (0.5 \times \rho \times Cd \times A \times (V_c^2 V_b - 2V_b^2 V_c + V_b^3)) \]

To find the maximum Power this expression is differentiated and set equal to zero

\[ \frac{d\text{Power}}{dV_b} = (0.5 \times \rho \times Cd \times A) \times (V_c^2 - 4V_b V_c + 3V_b^2) \]

\[ 0.5 \times \rho \times Cd \times A \neq 0 \]

\[ V_c^2 - 4V_b V_c + 3V_b^2 = 0 \]

\[ V_b = 1 \times V_c \]

\[ V_b = 1/3 \times V_c \]

By solving the quadratic two values of Vb are found- one when Vb = Vc (the wheel is freewheeling at this point) and one when Vb = 1/3 Vc, which has been used in these calculations.