**Converting Atmospheric Pollutant Concentrations**: from mg/m\(^3\) to ppmv

The conversion factor depends on the temperature at which you want the conversion (usually about 20 to 25 degrees Centigrade). At an ambient pressure of 1 atmosphere, the general equation is:

\[
\text{ppmv} = \frac{(\text{mg/m}^3)(273.15 + \degree\text{C})}{(12.187)(\text{MW})}
\]

where:

- ppmv = ppm by volume (i.e., volume of gaseous pollutant per 10\(^6\) volumes of ambient air)
- mg/m\(^3\) = milligrams of gaseous pollutant per cubic meter of ambient air
- MW = molecular weight of the gaseous pollutant
- °C = ambient air temperature in degrees Centigrade

As an example, for gaseous pollutant NOx, convert 20 mg/m\(^3\) to ppmv at 25 °C:

\[
\text{ppmv} = \frac{(20)(273.15 + 25)}{(12.187)(46.01)} = 10.6
\]

where: 46.01 = molecular weight of NO\(_2\) (i.e., NOx expressed as nitrogen dioxide)
Converting Atmospheric Pollutant Concentrations: from ppmv to mg/m³

The conversion factor depends on the temperature at which you want the conversion (usually about 20 to 25 degrees Centigrade). At an ambient pressure of 1 atmosphere, the general equation is:

\[ \text{mg/m}^3 = \frac{(\text{ppmv})(12.187)\text{MW}}{(273.15 + ^\circ \text{C})} \]

where:
- \( \text{mg/m}^3 \) = milligrams of gaseous pollutant per cubic meter of ambient air
- \( \text{ppmv} \) = ppm by volume (i.e., volume of gaseous pollutant per 10⁶ volumes of ambient air)
- \( \text{MW} \) = molecular weight of the gaseous pollutant
- \( ^\circ \text{C} \) = ambient air temperature in degrees Centigrade

As an example, for gaseous pollutant NOx, convert 20 ppmv to mg/m³ at 25 °C:

\[ \text{mg/m}^3 = \frac{(20)(12.187)(46.01)}{(273.15 + 25)} = 37.6 \]

where: 46.01 = molecular weight of NO₂ (i.e., NOx expressed as nitrogen dioxide)

NOTES:
1. The pollution laws and regulations in the United States typically reference their pollutant limits to an ambient temperature of 20 to 25 °C as noted above. However, in other nations, the reference ambient temperature for pollutant limits may be 0 °C or other values.
2. 1 percent by volume = 10,000 ppmv (i.e., parts per million by volume).
3. For all practical purposes, degrees Centigrade and degrees Celsius are synonymous.
Atmospheric pollutant concentrations expressed as mass per unit volume of atmospheric air (e.g., mg/m$^3$, ug/m$^3$, etc.) at sea level will decrease with increasing altitude because the atmospheric pressure decreases with increasing altitude.

The change of atmospheric pressure with altitude can be obtained from this equation:

$$ P_a = 0.9877^a $$

Given an atmospheric pollutant concentration at an atmospheric pressure of 1 atmosphere (i.e., at sea level altitude), the concentration at other altitudes can be obtained from this equation:

$$ C_a = (C)(0.9877^a) $$

where:

- $a$ = altitude, in 100's of meters
- $P_a$ = atmospheric pressure at altitude $a$, in atmospheres
- $C$ = concentration at sea level altitude, in mass per unit volume
- $C_a$ = concentration at altitude $a$, in mass per unit volume

As an example, given a concentration of 260 mg/m$^3$ at sea level, calculate the equivalent concentration at an altitude of 1,800 meters:

$$ C_a = (260)(0.9877^{18}) = 208 \text{ mg/m}^3 \text{ at 1,800 meters altitude} $$

Calculation Of Gas Densities:

The following equation for the density of a gas in pounds per cubic foot is derived from the ideal gas law and the applicable universal gas constant:

$$ \text{pounds per cubic foot} = \left( \frac{1}{Z} \right) \left( \frac{\text{MW}}{10.73} \right) \left( \frac{\text{psia}}{\circ\text{R}} \right) $$

The following equations for the density of a gas in kilograms per cubic meter are also derived from the ideal gas law and the applicable universal gas constants ... one of the equations uses the absolute pressure expressed in atmospheres and the other uses the absolute pressure expressed in kilopascals:
Conversions and Formulas

\[
\text{kilograms per cubic meter} = \left( \frac{1}{Z} \right) \left( \frac{MW}{0.082057} \right) \left( \frac{\text{atm}}{\degree K} \right)
\]

\[
\text{kilograms per cubic meter} = \left( \frac{1}{Z} \right) \left( \frac{MW}{8.3144} \right) \left( \frac{\text{kPa}}{\degree K} \right)
\]

where:
- \( Z \) = gas compressibility factor at the given temperature and pressure (dimensionless)
- \( MW \) = molecular weight of the gas
- \( \text{psia} \) = absolute pressure in pounds per square inch
- \( \text{atm} \) = absolute pressure in atmospheres
- \( \text{kPa} \) = absolute pressure in kilopascals
- \( 1 \text{ atm} = 14.696 \text{ psia} = 101.325 \text{ kPa} \)
- \( \degree R \) = absolute temperature of the gas in degrees Rankine = 459.67 + \( \degree F \)
- \( \degree K \) = absolute temperature of the gas in degrees Kelvin = 273.15 + \( \degree C \)

\( \degree F = \text{degrees Fahrenheit} = (1.8)(\degree C) + 32 \)
\( \degree C = \text{degrees Centigrade} = (\degree F - 32) / 1.8 \)

The numbers 10.73, 0.082057, and 8.3144 are all the universal gas law constant expressed in the applicable units for each of the above equations. For all practical purposes, degrees Centigrade and degrees Celsius are synonymous. Also, in many cases, it may be assumed that the ideal gas law applies and thus \( Z \) may be taken to be 1.00.

The technical literature can be very confusing because many authors fail to explain whether they are using the universal gas law constant \( R \) which applies to any ideal gas or whether they are using the gas law constant \( R_s \) which only applies to a specific individual gas. The relationship between the two constants is \( R_s = R / (MW) \).

Standard Conditions For Gas Volumes:

A normal cubic meter (Nm\(^3\)) is the metric expression of gas volume at standard conditions and it is usually defined as being measured at 0 °C and 1 atmosphere of pressure.

A standard cubic foot (scf) is the USA expression of gas volume at standard conditions and it is very often defined as being measured at 60 °F and 1 atmosphere of pressure. There are other definitions of standard gas conditions used in the USA besides 60 °F and 1 atmosphere, but that is the most common one ... and it is very widely used in the oil, gas and hydrocarbon processing industries.
That being understood:

1 Nm\(^3\) of any gas (measured at 0 °C and 1 atm. pressure) equals 37.326 scf of that gas (measured at 60 °F and 1 atm. pressure) ... and thus 1 Nm\(^3\) per hour of any gas equals 0.622 scf per minute of that gas.

1 kg-mol of any ideal gas equals 22.414 Nm\(^3\) of that gas ... and 1 lb-mol of any ideal gas equals 379.482 scf of that gas.

**Gas Volume Conversions:**

To convert air or other gas volumes from one pressure (P\(_1\)) and temperature (T\(_1\)) to another pressure (P\(_2\)) and temperature (T\(_1\)), use the following equation:

\[
\frac{V_2}{V_1} = \left( \frac{Z_2}{Z_1} \right) \left( \frac{P_1}{P_2} \right) \left( \frac{T_2}{T_1} \right)
\]

where:

- \(Z_1\) and \(Z_2\) = gas compressibility factors (which are non-dimensional)
- \(V_1\) and \(V_2\) = gas volumes in the same dimensional units
- \(P_1\) and \(P_2\) = absolute pressures in the same dimensional units
- \(T_1\) and \(T_2\) = absolute temperatures in the same units (either degrees °R or degrees °K)

\[°R = \text{absolute temperature in degrees Rankine} = 459.67 + °F\]
\[°K = \text{absolute temperature in degrees Kelvin} = 273.15 + °C\]
\[°F = \text{degrees Fahrenheit} = (1.8) (°C) + 32\]
\[°C = \text{degrees Centigrade} = (°F - 32) / 1.8\]

For all practical purposes, degrees Centigrade and degrees Celsius are synonymous. Also, in many cases, it may be assumed that the ideal gas law applies and thus Z may be taken to be 1.00.

**Definition Of The Pasquill Stability Classes:**
The amount of turbulence in the ambient air has a major effect upon the rise and dispersion of air pollutant plumes. The amount of turbulence can be categorized into defined increments or "stability classes". The most commonly used categories are the Pasquill stability classes A, B, C, D, E, and F. Class A denotes the most unstable or most turbulent conditions and Class F denotes the most stable or least turbulent conditions.

The Pasquill stability classes are presented below as they are defined by the prevailing meteorological conditions of: (a) surface windspeed measured at 10 meters above ground level and (b) day-time incoming solar radiation or the night-time percentage of cloud cover.

<table>
<thead>
<tr>
<th>Surface Windspeed</th>
<th>Daytime Incoming Solar Radiation</th>
<th>Night-time Cloud Cover</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Strong</td>
<td>Moderate</td>
</tr>
<tr>
<td>m/s</td>
<td>mi/hr</td>
<td></td>
</tr>
<tr>
<td>&lt;2</td>
<td>&lt;5</td>
<td>A</td>
</tr>
<tr>
<td>2-3</td>
<td>5-7</td>
<td>A-B</td>
</tr>
<tr>
<td>3-5</td>
<td>7-11</td>
<td>B</td>
</tr>
<tr>
<td>5-6</td>
<td>11-13</td>
<td>C</td>
</tr>
<tr>
<td>&gt;6</td>
<td>&gt;13</td>
<td>C</td>
</tr>
</tbody>
</table>

Note: Class D applies to heavily overcast skies, at any windspeed day or night.

NOTES:
(1) m/s = meters per second
(2) mi/hr = statute miles per hour

Pressure Conversions:

Atmospheric pressures may be expressed in a number of different units. The following table provides the conversions between six of the most commonly used units of pressure. Here are some examples as read from the table:

(1) 1 atmosphere = 14.696 psi or 101.325 kPa
(2) 1 bar = 0.986923 atmospheres or 750.0616 mm Hg
(3) 1 psi = 6.894733 kPa or 51.71475 mm Hg
Conversions and Formulas

<table>
<thead>
<tr>
<th></th>
<th>1 kPa</th>
<th>0.010000</th>
<th>7.500617</th>
<th>0.010197</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 bar</td>
<td>0.986923</td>
<td>14.50382</td>
<td>100.0000</td>
<td>1</td>
</tr>
<tr>
<td>1 mm Hg</td>
<td>0.001316</td>
<td>0.019337</td>
<td>0.133322</td>
<td>0.001333</td>
</tr>
<tr>
<td>1 kg/cm²</td>
<td>0.967841</td>
<td>14.22339</td>
<td>98.0665</td>
<td>0.980665</td>
</tr>
</tbody>
</table>

Although not included in the above table:

- 1 atmosphere of pressure = 33.90 feet of water = 10.33 meters of water
- 1 mm Hg = 1 torr

Notes:

atm = absolute pressure, in atmospheres
psi = absolute pressure, in pounds per square inch
kPa = absolute pressure, in kilopascals
bar = absolute pressure, in bars
mm Hg = absolute pressure, in millimeters of Mercury
kg/cm² = absolute pressure, in kilograms per square centimeter

Effect of Altitude on Windspeeds:

The winds aloft generally have a higher velocity than the winds at ground level. In other words, at any given time and place, windspeed usually increases with altitude. The effect of altitude on windspeed involves two factors:

- the degree of turbulent mixing prevailing in the atmosphere at the given time and place, as characterized by the Pasquill stability class
- the terrain's surface area roughness, which induces surface friction at the given place

It has generally been agreed that the effect of altitude on windspeed is logarithmic and can be expressed as:

\[
\frac{u_z}{u_g} = \left(\frac{h_z}{h_g}\right)^n
\]

where:

\[
\frac{u_z}{u_g} = \left(\frac{h_z}{h_g}\right)^n
\]
Conversions and Formulas

\[ u_z = \text{wind velocity at height } z \]
\[ u_g = \text{wind velocity at ground station height} \]
\[ h_z = \text{height } z \]
\[ h_g = \text{ground station height (usually 10 meters)} \]
\[ n = \text{a function of the Pasquill stability class and the terrain type (see tables below)} \]

<table>
<thead>
<tr>
<th>Stability</th>
<th>Exponent n</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.10</td>
</tr>
<tr>
<td>B</td>
<td>0.15</td>
</tr>
<tr>
<td>C</td>
<td>0.20</td>
</tr>
<tr>
<td>D</td>
<td>0.25</td>
</tr>
<tr>
<td>E</td>
<td>0.25</td>
</tr>
<tr>
<td>F</td>
<td>0.30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stability</th>
<th>Exponent n</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.15</td>
</tr>
<tr>
<td>B</td>
<td>0.20</td>
</tr>
<tr>
<td>C</td>
<td>0.25</td>
</tr>
<tr>
<td>D</td>
<td>0.40</td>
</tr>
<tr>
<td>E</td>
<td>0.60</td>
</tr>
</tbody>
</table>

As an example, given a windspeed of 5 m/s measured at 10 meters above the ground and a stability class of B in rural terrain, calculate the windspeed at 500 meters above ground:

\[ u_z = (5)(500/10)^{0.15} = 9 \text{ m/s} \]

Converting Mass Flow Rates To Volumetric Flow Rates:

Gaseous emission flow rates (from process vents, combustion flue gases from furnaces or boilers, accidental gaseous releases, etc.) are often expressed in mass flow rates. To convert such mass flow rates to volumetric flow rates, first calculate the gas density (as explained in one of the sections above) using the actual temperature and pressure of the gaseous emission. Then use either of the following equations:

\[ \text{ft}^3 / \text{hr} = (\text{lbs} / \text{hr}) / (\text{lbs} / \text{ft}^3) \]

where:

\[ \text{ft}^3 / \text{hr} = \text{gas volumetric flow rate in cubic feet per hour} \]
\[ \text{lbs} / \text{hr} = \text{gas mass flow rate in pounds per hour} \]
\[ \text{lbs} / \text{ft}^3 = \text{gas density in pounds per cubic foot} \]
Conversions and Formulas

\[ m^3/\text{hr} = (\text{kg/\text{hr}}) / (\text{kg/m}^3) \]

where:
- \[ m^3/\text{hr} \] = gas volumetric flow rate in cubic meters per hour
- \[ \text{kg/\text{hr}} \] = gas mass flow rate in kilograms per hour
- \[ \text{kg/m}^3 \] = gas density in kilograms per cubic meter

Note: When calculating the density of the gaseous emission, the actual pressure of the gaseous emission at the point where it exits from the source vent or flue gas stack is taken as 14.696 psia or 1 atmosphere.

Windspeed Conversion Factors:

Meteorological data includes windspeeds which may be expressed as statute miles per hour, knots, or meters per second. Here are the conversion factors for those various expressions of windspeed:

1 knot = 1.152 statute mi/hr = 0.515 m/sec

1 statute mi/hr = 0.868 knots = 0.447 m/sec

1 m/sec = 2.237 statute mi/hr = 1.942 knots

Note:
1 statute mile = 5,280 feet = 1,609 meters

Correcting Concentrations to Reference Conditions in Regulated Emission Limits:

Many environmental protection agencies have issued regulations that limit the concentration of pollutants in gaseous emissions and define the reference conditions applicable to those concentration limits. For example, such a regulation might limit the concentration of NOx to 55 ppmv in a dry combustion exhaust gas corrected to 3 volume percent O\textsubscript{2}. As another example, a regulation might limit the concentration of particulate matter to 0.1 grain per standard cubic foot (i.e., scf) of dry exhaust gas corrected to 12 volume percent CO\textsubscript{2}. 

A standard cubic foot of dry gas is often denoted as "dscf" or as "scfd". Likewise, a standard cubic meter of dry gas is often denoted as "dscm" or "scmd" by environmental agencies in the USA.

**Correcting Concentrations to a Dry Basis:**

If a gaseous emission sample is analyzed and found to contain water vapor and a pollutant concentration of $X$, then $X$ should be designated as the "wet basis" pollutant concentration. The following equation can be used to correct the measured "wet basis" concentration to a "dry basis" concentration:

$$
\text{dry basis concentration} = \left( \frac{\text{wet basis concentration}}{1 - w} \right)
$$

where:

$$w = \text{fraction of the emitted exhaust gas, by volume, which is water vapor}$$

Thus, a wet basis concentration of 40 ppmv in an emitted gas containing 10 volume percent water vapor would have a dry basis concentration $= \left( \frac{40}{1 - 0.10} \right) = 44.44$ ppmv.

**Correcting Concentrations to a Reference $O_2$ Content in the Emitted Gas:**

The following equation can be used to correct a measured pollutant concentration in an emitted gas (containing a measured $O_2$ content) to an equivalent pollutant concentration in an emitted gas containing a specified reference amount of $O_2$:

$$
C_r = C_m \left( \frac{20.9 - r}{20.9 - m} \right)
$$

where:

$$C_r = \text{corrected concentration in dry emitted gas having the reference volume } % \ O_2 = r$$
$$C_m = \text{measured concentration in dry emitted gas having the measured volume } % \ O_2 = m$$

Thus, a measured nitrogen oxides (i.e., NOx) concentration of 45 ppmv (dry basis) in an emitted gas having 5 volume $% \ O_2 = \left( \frac{45}{20.9 - 5} \right) = 50.7$ ppmv (dry basis) when corrected to an emitted gas having a specified reference $O_2$ content of 3 volume $%$.

**Correcting Concentrations to a Reference $CO_2$ Content in the Emitted Gas:**

The following equation can be used to correct a measured pollutant concentration in an emitted
gas (containing a measured CO₂ content) to an equivalent pollutant concentration in an emitted gas containing a specified reference amount of CO₂:

\[
(3) \quad C_r = C_m \left( \frac{r}{m} \right)
\]

where:

- \( C_r \) = corrected concentration in dry emitted gas having the reference volume % CO₂ = \( r \)
- \( C_m \) = measured concentration in dry emitted gas having the measured volume % CO₂ = \( m \)

And thus, a measured particulate matter concentration of 0.1 grain per dscf in an emitted gas that has 8 volume % CO₂ = \( (0.1) \left( \frac{12}{8} \right) \) = 0.15 grain per dscf when corrected to an emitted gas having a specified reference CO₂ content of 12 volume %.

Notes:
- Although ppmv and grains per dscf have been used in the above examples, you may use other concentrations such as ppbv (i.e., parts per billion by volume), volume percent, grams per dscm, etc.
- 1 percent by volume = 10,000 ppmv (i.e., parts per million by volume).
- Equation (1) above is from "40 CFR, Chapter I, Part 60, Appendix A-3, Test Method 4".
- Equation (2) above is from "40 CFR, Chapter I, Part 60, Appendix B, Performance Spec. 2".
- Equation (3) above is from "40 CFR, Chapter I, Part 60".

Exhaust Gas Generated From Combustion of Fuels:

It is often useful to have a good estimate of the amount of exhaust gas or flue gas generated by the combustion of a fuel and of the O₂ and CO₂ content of the gas. Here are some typical values:
### Conversions and Formulas

<table>
<thead>
<tr>
<th>Property</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Higher heating value, Btu / scf</td>
<td>1,093</td>
<td>150,000</td>
<td>8,020</td>
</tr>
<tr>
<td>Higher heating value, Btu / gallon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Higher heating value, Btu / pound</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Molecular weight</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gravity, °API</td>
<td>18</td>
<td>15.5</td>
<td></td>
</tr>
<tr>
<td>Carbon / hydrogen ratio by weight</td>
<td></td>
<td></td>
<td>8.1</td>
</tr>
<tr>
<td>Weight % carbon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight % hydrogen</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight % oxygen</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight % sulfur</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight % nitrogen</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight % ash</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight % moisture</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Excess combustion air, %</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Amount of wet exhaust gas, scf / MMBtu of fuel</td>
<td>11,600</td>
<td>11,930</td>
<td>13,985</td>
</tr>
<tr>
<td>CO₂ in wet exhaust gas, volume %</td>
<td>8.8</td>
<td>12.4</td>
<td>13.5</td>
</tr>
<tr>
<td>O₂ in wet exhaust gas, volume %</td>
<td>2.0</td>
<td>2.6</td>
<td>3.3</td>
</tr>
<tr>
<td>Molecular weight of wet exhaust gas</td>
<td>9,510</td>
<td>10,600</td>
<td>12,130</td>
</tr>
<tr>
<td>Amount of dry exhaust gas, scf / MMBtu of fuel</td>
<td>10.8</td>
<td>14.0</td>
<td>15.5</td>
</tr>
<tr>
<td>CO₂ in dry exhaust gas, volume %</td>
<td>2.5</td>
<td>2.9</td>
<td>3.8</td>
</tr>
<tr>
<td>O₂ in dry exhaust gas, volume %</td>
<td>29.9</td>
<td>30.4</td>
<td>30.8</td>
</tr>
</tbody>
</table>

### Converting the Exhaust Gas Amounts to Other Units:

The amount of fuel combusted may be expressed in MMBtu, or in MMkcal, or in MW-hr ... and the amount of combustion exhaust gas may be expressed as standard cubic feet (scf) or as Normal cubic meters (Ncm or Nm³). These are the definitions and equivalents involved in converting the exhaust gas amounts from scf / MMBtu to other units:

(a) 1 MMBtu = 10^6 Btu
(b) 1 MMkcal = 10^6 kilogram-calories
(c) 1 MW-hr = 1 megawatt-hour = 10^6 watt-hours
(d) 1 MMBtu = 0.252 MMkcal = 0.293 MW-hr
(e) 1 MMkcal = 3.968 MMBtu = 1.163 MW-hr
(f) 1 MW-hr = 3.413 MMBtu = 0.860 MMkcal
Conversions and Formulas

(g) scf = standard cubic feet measured at 60 °F and atmospheric pressure
(h) Nm³ = Normal cubic meters measured at 0 °C and atmospheric pressure
(i) 1 Nm³ = 37.326 scf

These are the resulting conversions from scf / MMBtu to other units:

\[ 1 \text{ scf} / \text{MMBtu} = 0.1063 \text{ Nm}^3 / \text{MMkcal} = 0.0914 \text{ Nm}^3 / \text{MW-hr} \]

Notes:

-- Reference temperatures (other than those above) are used to define standard cubic feet and Normal cubic meters, but those given above are widely used. As an example, the USA's Environmental Protection Agency uses 68 °F (20 °C) as the reference temperature for both standard cubic feet (scf) and standard cubic meters (scm) ... whereas most of the oil and gas industries worldwide generally use 60 °F to define scf, and all of the metric nations use the term Normal m³ (rather than scm) with a reference temperature of 0 °C (rather than 20 °C).

-- The dry and wet exhaust gas volumes given above differ somewhat from the U.S. EPA's corresponding F Factors (see publication EPA-454/R-95-015, Revised) because: (a) the EPA's F Factors are all at 0% excess combustion air, (b) the EPA's reference temperature for scf is 68 °F rather than the 60 °F used above, and (c) the fuel compositions that were used for the EPA's F Factors probably differ from those used above.

-- In the case of MW-hr, the M is an abbreviation for 10⁶ ... whereas in the case of MMBtu and MMkcal, M is an abbreviation for 10³. This is an unfortunate irregularity, but one which is actually in use.

Conversion Factors and Dimensional Analysis:

Every physical measurement consists of a numerical quantity and a corresponding dimensional unit (for example: 1000 kg / m³, 50 miles / hour, 1000 Btu / lb, etc). Whenever it is necessary to convert a physical measurement from one dimensional unit to another, dimensional analysis (also known as the unit-factor method or the factor-label method) is quite useful.

But what is dimensional analysis in the context of converting dimensional units? It is the sequential usage of various conversion factors expressed as fractions and arranged so that any dimensional unit appearing in both the numerator and denominator of any of the fractions can be cancelled out until only the desired set of dimensional units is obtained. For example, let us convert 10 miles per hour to meters per second:
Conversions and Formulas

\[
\begin{array}{ccc}
10 \text{ mile} & 1609 \text{ meter} & 1 \text{ hour} & 4.47 \text{ meter} \\
\hline
1 \text{ hour} & 1 \text{ mile} & 3600 \text{ second} & \text{second}
\end{array}
\]

As can be seen, when the mile dimensions and the hour dimensions are cancelled out and the arithmetic is done, we have converted 10 miles per hour to 4.47 meters per second.

As another example, convert the ppmv of NOx in an exhaust gas (denoted as C) to grams per hour of NOx, given the molecular weight of NOx as 46.01, and the rate of exhaust in scf per minute (denoted as E):

\[
\frac{\text{C, scf}}{10^6 \text{ scf exhaust}} = \frac{\text{E, scf exhaust}}{} \\
\frac{\text{grams}}{1 \text{ minute}} = \frac{60 \text{ minute}}{1 \text{ hour}} \frac{46.01 \text{ lb}}{453.6 \text{ grams}} \frac{1 \text{ lb-mole}}{1 \text{ lb-mole}} \frac{\text{1 lb}}{379.48 \text{ scf}}
\]

As shown in the above equation, after cancelling out any dimensional units that appear both above and below the division lines, the only remaining units are grams / hour. Thus:

\[
\text{grams / hour of NOx} = \frac{(C, \text{ ppmv of NOx}) \times (E, \text{ scf of exhaust / minute})}{303.05}
\]

Note: A standard cubic foot (scf) is the USA expression of gas volume at 60 °F and 1 atmosphere of pressure and, using that definition, there are 379.48 scf per pound-mole of any gas. There are other definitions of standard gas conditions used in the USA besides 60 °F and 1 atmosphere, but this is the most common one.

The same example, using metric units:

\[
\frac{\text{C, Nm}^3}{10^6 \text{ Nm}^3 \text{ exhaust}} = \frac{\text{E, Nm}^3 \text{ exhaust}}{1 \text{ minute}} \frac{60 \text{ minute}}{1 \text{ hour}} \frac{46.01 \text{ grams}}{1 \text{ g-mole}} \frac{1 \text{ g-mole}}{0.02214 \text{ Nm}^3}
\]

As shown in the above equation, after cancelling out any dimensional units that appear both above and below the division lines, the only remaining units are grams / hour. Thus:

\[
\text{grams / hour of NOx} = \frac{(C, \text{ ppmv of NOx}) \times (E, \text{ Nm}^3 \text{ of exhaust / minute})}{8.12}
\]

Note: A normal cubic meter (Nm³) is the usual metric system expression of gas volume at 0 °C and 1 atmosphere of pressure and, using that definition, there are 0.02214 Nm³ per gram-mole of any gas.

Dimensional analysis can also be used to check the correctness of any mathematical equation involving dimensional units by checking to see that the dimensional units on the left hand side of the equation are the same as the dimensional units on the right hand side of the equation.
Appendix A: Concentration Units: (ppm and mg/m³)

1. The term ppm is an acronym for parts per million. In the context of airborne gaseous pollutant concentrations, it stands for volumes of gaseous pollutant X per million volumes of air. As discussed further below, it is very important to use the terms ppmv or ppm by volume rather than simply ppm.

Some airborne gaseous pollutant concentrations may be expressed as ppbv or ppb by volume, meaning volumes of gaseous pollutant per billion volumes of air.

2. For an airborne gaseous or non-gaseous pollutant concentration, the term mg/m³ stands for milligrams of substance X per cubic meter of air.

Some airborne gaseous or non-gaseous pollutant concentrations may be expressed as ug/m³, which stands for micrograms of substance X per cubic meter of air.

3. The hazardous concentration limits set by NIOSH, OSHA and ACGIH are for the most part directed at airborne pollutants (i.e., gases, vapors, dusts, aerosols, and mists). Hazardous substances dissolved in water or any other liquid are generally not within the purview of NIOSH, OSHA and ACGIH.

4. Airborne pollutant concentration limits are usually expressed as parts per million by volume (i.e., ppmv) for gases and vapors, and mg/m³ for dusts, aerosols and mists.

5. Quite often you will find the hazardous concentration limit of an airborne gaseous pollutant expressed as either ppmv or mg/m³ or both. There is a simple mathematical method of converting one to the other for gaseous substances (as given earlier in this article). However, for non-gaseous pollutants such as dusts, mists or aerosols, it would be very difficult, if not impossible, to convert mg/m³ to ppmv.

6. When dealing with hazardous substances dissolved in water or any other liquid, most chemists would use the term parts per million by weight (i.e., ppmw) as meaning weight of dissolved substance X per million weights of liquid ... where the weight units might be in milligrams (mg) or grams (g) or pounds (lb) or kilograms (kg).

In a few cases, chemists might use ppmv as meaning volumes of gas or liquid dissolved in water or other liquid per million volumes of water or other liquid. For example, gaseous carbon dioxide dissolved in water ... or liquid acetone dissolved in water ... or liquid additive dissolved in gasoline.

7. The numerical difference between a concentration expressed as ppmv or ppmw can be very large, especially for gaseous substances. Thus, it is most important to be as specific as possible and
to use the terms ppmv or ppmw rather than simply ppm. Confusion as to whether ppm means ppmv or ppmw can have serious consequences. It is also important to state whether you are dealing with substances in the air or substances within water or other liquid.

(8) Finally, keep in mind that if something can be mis-construed, it will happen. Be as specific as possible in defining concentration limits. If you find an exception to what is said above, it is either valid because of some special or unusual reason, or the exception is simply incorrect.